Uniwersytet Wrocławski

Wrocław, 24 października 2014 r.

Sesja poświęcona działalności prof. Stanisława Lewanowicza

- Powitanie gości, prof. Leszek Pacholski
- Wystąpienie prof. Stefana Paszkowskiego
- *Staszek i historia Instytutu na tle historii komputerów i informatyki*, prof. Maciej M. Sysło
- *Działalność naukowa prof. Stanisława Lewanowicza*, dr hab. Paweł Woźny

Po sesji zapraszamy na poczęstunek do sali 141 im. Jerzego Szczepkowicza.

Działalność naukowa prof. Stanisława Lewanowicza

Paweł Woźny

Wrocław, 24 października 2014 r.

Działalność naukowa prof. Stanisława Lewanowicza
(w telegraficznym skrócie)

Paweł Woźny

Wrocław, 24 października 2014 r.

Zainteresowania naukowe prof. Lewanowicza

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- Równania rekurencyjne i ich zastosowania

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- Równania rekurencyjne i ich zastosowania
- Funkcje hipergeometryczne

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- Równania rekurencyjne i ich zastosowania
- Funkcje hipergeometryczne
- Wielomiany ortogonalne

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- Metody matematyczne modelowania krzywych i powierzchni
- Metody numeryczne

Równania rekurencyjne i ich zastosowania

Problem (S. Paszkowski). Na podstawie równania różniczkowego o wielomianowych współczynnikach spełnianego przez funkcję f znaleźć związek rekurencyjny najniższego możliwego rzędu zachodzący dla współczynników Gegenbauera $c_k[f]$ tej funkcji

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$$f(x) = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)}(x)$$

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$$f(x) = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)}(x)$$

$$\Downarrow$$

$$\sum_{i=0}^{m} p_i(x) f^{(i)}(x) = g(x)$$

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$$\sum_{j=0}^{r} A_j(k) c_{j+k}[f] = B(k), \quad r \text{ – minimalne!}$$

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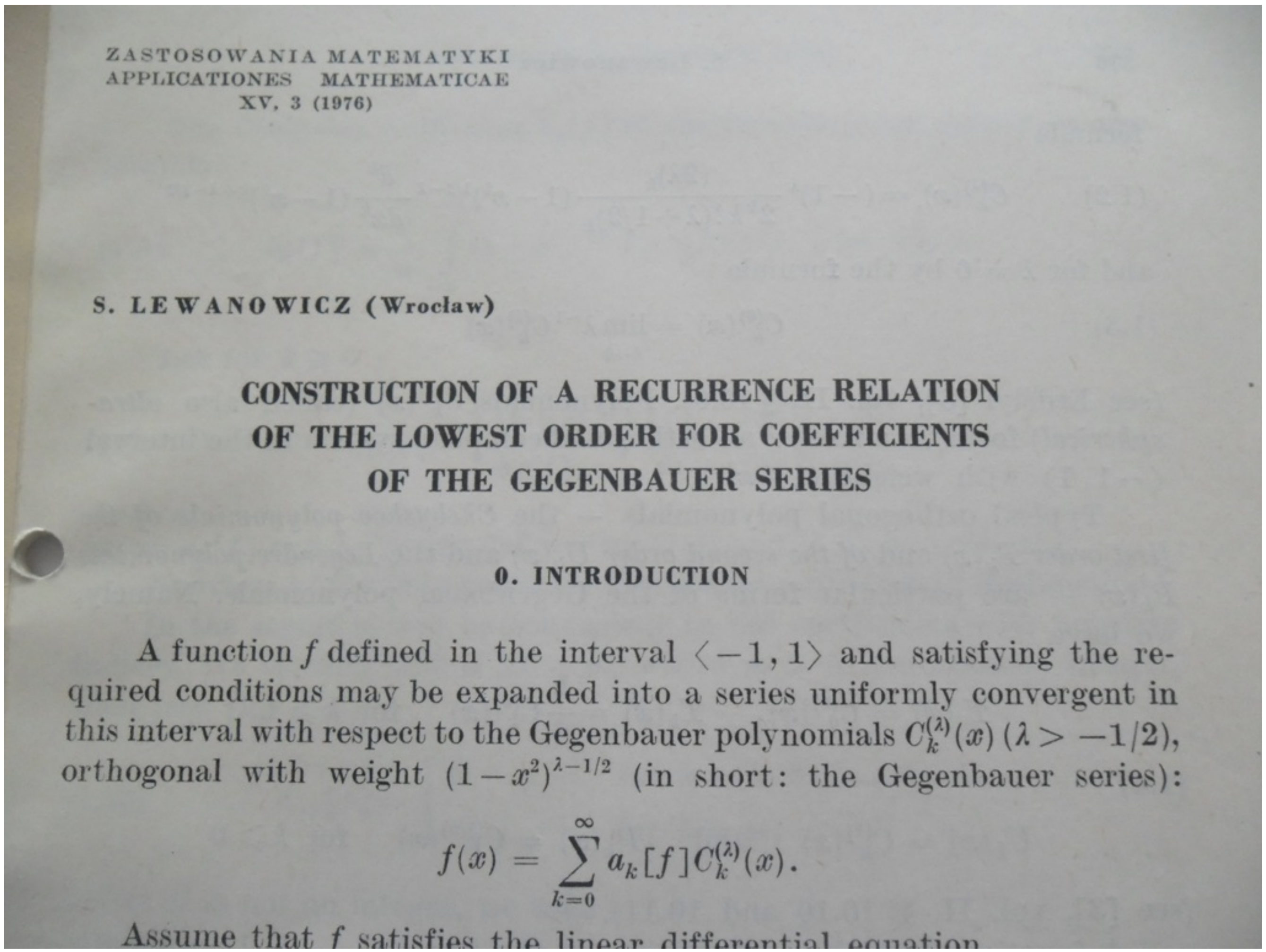
Zastosowania

- rozwiązywanie równań różniczkowych
- całkowanie numeryczne
- wyznaczanie i badanie rozwinięć ortogonalnych

Równania rekurencyjne i ich zastosowania

$$f = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)} \quad \Rightarrow \quad \sum_{i=0}^{m} p_i f^{(i)} = g \quad \Rightarrow \quad \sum_{j=0}^{r} A_j(k) c_{j+k}[f] = B(k), \quad r - \text{min!}$$

Rozwiązanie (1975)



ZASTOSOWANIA MATEMATYKI
APPLICATIONES MATHEMATICAE
XV, 3 (1976)

S. LEWANOWICZ (Wrocław)

CONSTRUCTION OF A RECURRENCE RELATION OF THE LOWEST ORDER FOR COEFFICIENTS OF THE GEGENBAUER SERIES

0. INTRODUCTION

A function f defined in the interval $\langle -1, 1 \rangle$ and satisfying the required conditions may be expanded into a series uniformly convergent in this interval with respect to the Gegenbauer polynomials $C_k^{(\lambda)}(x)$ $(\lambda > -1/2)$, orthogonal with weight $(1-x^2)^{\lambda-1/2}$ (in short: the Gegenbauer series):

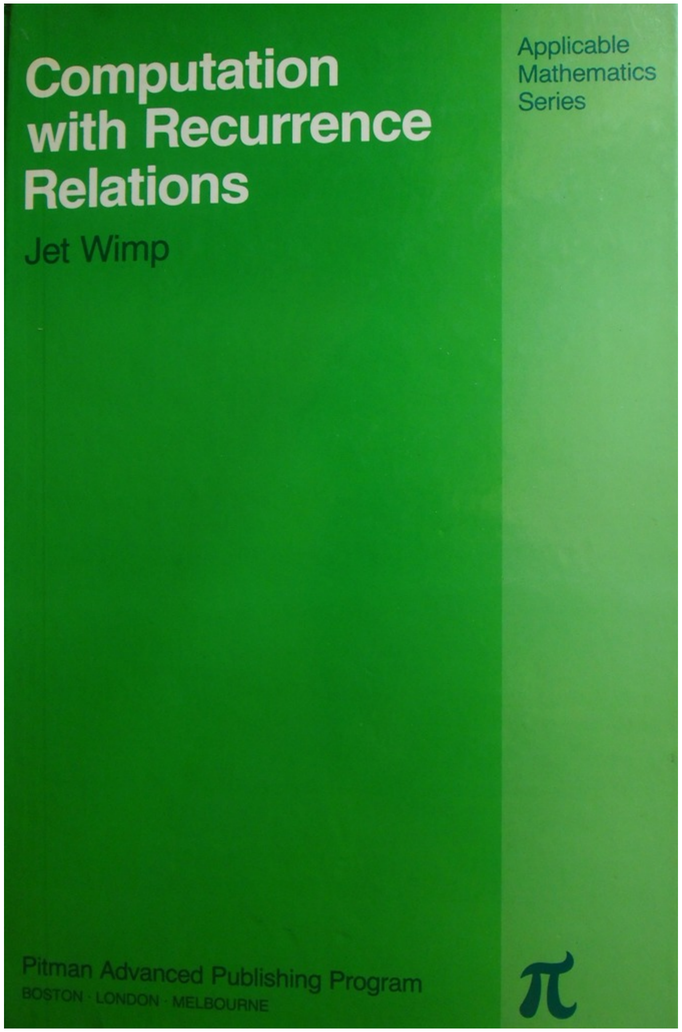
$$f(x) = \sum_{k=0}^{\infty} a_k[f] C_k^{(\lambda)}(x).$$

Assume that f satisfies the linear differential equation

Równania rekurencyjne i ich zastosowania

$$f = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)} \quad \Rightarrow \quad \sum_{i=0}^{m} p_i f^{(i)} = g \quad \Rightarrow \quad \sum_{j=0}^{r} A_j(k) c_{j+k}[f] = B(k), \quad r - \text{min!}$$

A co na to wielki Jet Wimp? (1984)



Równania rekurencyjne i ich zastosowania

$$f = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)} \quad \Rightarrow \quad \sum_{i=0}^{m} p_i f^{(i)} = g \quad \Rightarrow \quad \sum_{j=0}^{r} A_j(k) c_{j+k}[f] = B(k), \quad r - \text{min!}$$

A co na to wielki Jet Wimp? (1984)

11 Series solutions to ordinary differential equations

When one attempts to satisfy a linear differential equation with a series of the form

$$\sum C(n) p_n(z),$$

and the functions $\{p_n\}$ satisfy linear difference equations it is often found that the coefficients $C(n)$ satisfy linear *difference* equations. If the base functions $\{p_n\}$ are chosen appropriately so that the previous expansion converges rapidly then it is reasonable to expect that $C(n)$ will be a minimal solution of the difference equation it satisfies and therefore that it can be computed efficiently from the equation by the Miller or Olver algorithms or one of their variants. Such indeed is usually the case and research into this and related ideas has provided a highly effective body of techniques for approximating the solutions of differential equations, even nonlinear equations.

There are actually two topics of interest. The first is the derivation of the required difference equation for the coefficients $C(n)$. Work in this area has been done by many mathematicians, but the largest and most comprehensive theory is due to the Polish mathematician S. Lewanowicz. I discuss some of his findings in Section 11.2.3.

Równania rekurencyjne i ich zastosowania

$$f = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)} \quad \Rightarrow \quad \sum_{i=0}^{m} p_i f^{(i)} = g \quad \Rightarrow \quad \sum_{j=0}^{r} A_j(k) c_{j+k}[f] = B(k), \quad r - \text{min!}$$

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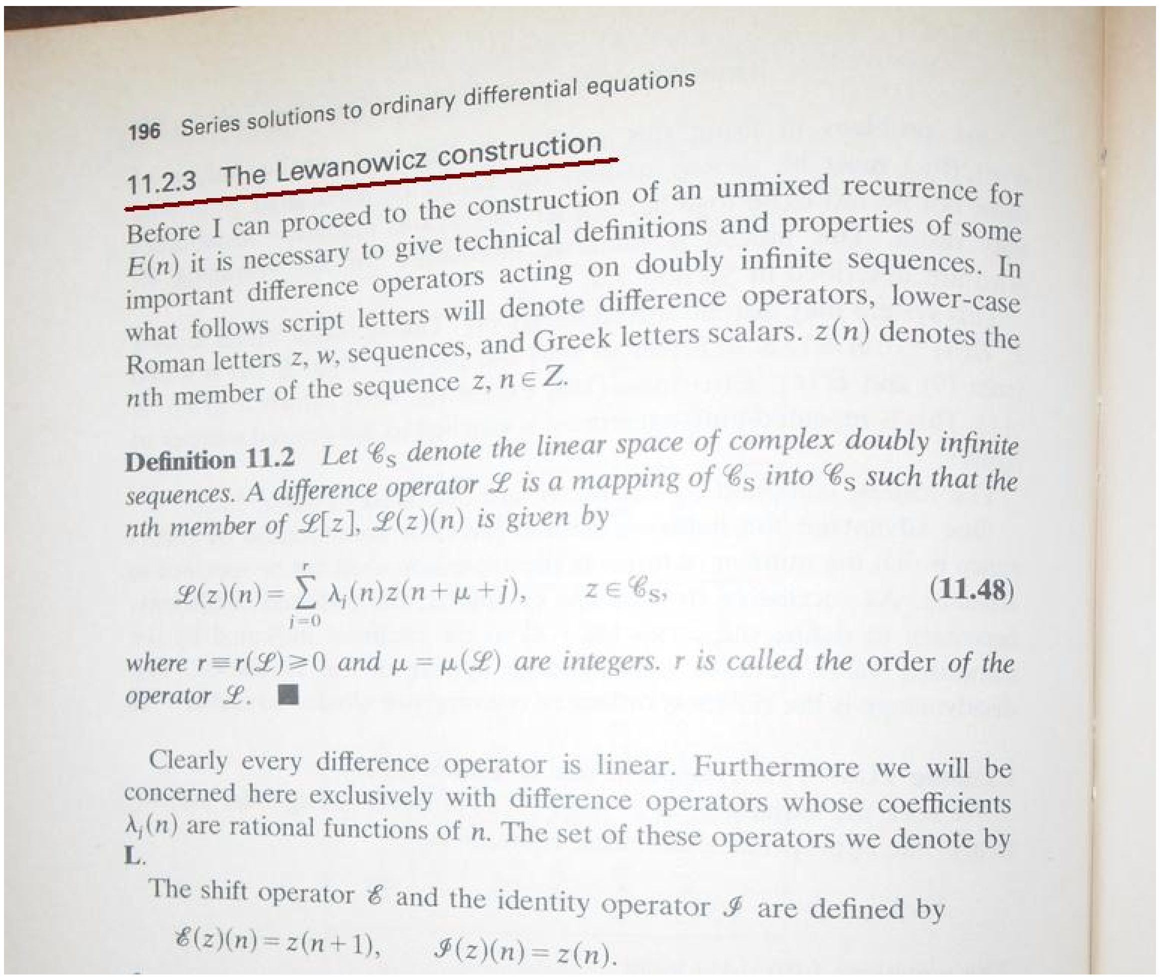

196 Series solutions to ordinary differential equations

11.2.3 The Lewanowicz construction

Before I can proceed to the construction of an unmixed recurrence for $E(n)$ it is necessary to give technical definitions and properties of some important difference operators acting on doubly infinite sequences. In what follows script letters will denote difference operators, lower-case Roman letters z, w, sequences, and Greek letters scalars. $z(n)$ denotes the nth member of the sequence z, $n \in Z$.

Definition 11.2 *Let $\mathscr{C}_S$ denote the linear space of complex doubly infinite sequences. A difference operator $\mathscr{L}$ is a mapping of $\mathscr{C}_S$ into $\mathscr{C}_S$ such that the nth member of $\mathscr{L}[z]$, $\mathscr{L}(z)(n)$ is given by*

$$\mathscr{L}(z)(n) = \sum_{j=0}^{r} \lambda_j(n) z(n+\mu+j), \qquad z \in \mathscr{C}_S, \tag{11.48}$$

where $r \equiv r(\mathscr{L}) \geqslant 0$ and $\mu = \mu(\mathscr{L})$ are integers. r is called the order *of the operator $\mathscr{L}$.* ■

Clearly every difference operator is linear. Furthermore we will be concerned here exclusively with difference operators whose coefficients $\lambda_j(n)$ are rational functions of n. The set of these operators we denote by **L**.

The shift operator $\mathscr{E}$ and the identity operator $\mathscr{I}$ are defined by

$$\mathscr{E}(z)(n) = z(n+1), \qquad \mathscr{I}(z)(n) = z(n).$$

Równania rekurencyjne i ich zastosowania

$$f = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)} \quad \Rightarrow \quad \sum_{i=0}^{m} p_i f^{(i)} = g \quad \Rightarrow \quad \sum_{j=0}^{r} A_j(k) c_{j+k}[f] = B(k), \quad r - \text{min!}$$

Sevilla, 1997

Równania rekurencyjne i ich zastosowania

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A 33 lata później Alexandre Benoit w swoim doktoracie... (2009)

CHEBYSHEV EXPANSIONS FOR
SOLUTIONS OF LINEAR DIFFERENTIAL EQUATIONS

ALEXANDRE BENOIT AND BRUNO SALVY

Abstract. A Chebyshev expansion is a series in the basis of Chebyshev polynomials of the first kind. When such a series solves a linear differential equation, its coefficients satisfy a linear recurrence equation. We interpret this equation as the numerator of a fraction of linear recurrence operators. This interpretation lets us give a simple view of previous algorithms, analyze their complexity, and design a faster one for large orders.

1. Introduction

Chebyshev series are series of the form

$$f(x) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n T_n(x), \tag{1}$$

where T_n denotes the nth Chebyshev polynomial of the first kind. These polynomials can be defined by

$$T_n(\cos\theta) = \cos(n\theta), \tag{2}$$

Nowa interpretacja i bardzo szybka implementacja algorytmu Lewanowicza

Równania rekurencyjne i ich zastosowania

$$f = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)} \quad \Rightarrow \quad \sum_{i=0}^{m} p_i f^{(i)} = g \quad \Rightarrow \quad \sum_{j=0}^{r} A_j(k) c_{j+k}[f] = B(k), \quad r - \text{min!}$$

A 33 lata później Alexandre Benoit w swoim doktoracie... (2009)

6 ALEXANDRE BENOIT AND BRUNO SALVY

Algorithm 1 Lewanowicz' algorithm

Input: $L := \sum_{i=0}^{k} p_i(x)\partial^i$
Output: (P, Q) such that $\varphi(L) = Q^{-1}P$
  $P := p_k(X)$
  $Q := 1$
  for all i **from** $k-1$ **to** 0 **do**
    Compute $\mathrm{lclm}((S^{-1} - S), P) = \hat{P}P = \hat{U}(S^{-1} - S)$.
    $Q := \hat{P}Q$
    $P := \hat{U}2n + Qp_i(X)$
  end for
  return (P, Q)

3.2. Horner's Rule and Lewanowicz' Algorithm.

Proposition 2. *Let $L = p_k(x)\partial_x^k + \cdots + \partial_x p_0(x)$ be a linear differential operator in $\mathbb{Q}[x]\langle\partial_x; \mathrm{Id}, d/dx\rangle$. The evaluation of $\varphi(L)$ by Horner's rule*

$$\varphi(L) = (\cdots(p_k(X)D + p_{k-1}(X))D + \cdots)D + p_0(X)$$

using Eqs. (11) and (12) for the computation of sums and products produces a fraction $Q^{-1}P$ that is irreducible.

Nowa interpretacja i bardzo szybka implementacja algorytmu Lewanowicza

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$$f = \sum_{k=0}^{\infty} c_k[f] C_k^{(\lambda)} \quad \Rightarrow \quad \sum_{i=0}^{m} p_i f^{(i)} = g \quad \Rightarrow \quad \sum_{j=0}^{r} A_j(k) c_{j+k}[f] = B(k), \quad r - \text{min!}$$

Pomysł Lewanowicza był nadal rozwijany. Doczekała się wielu odmian, uogólnień i zastosowań:

- *Construction of the lowest order recurrence relation for the Jacobi coefficients*, Zastosowania Matematyki 17 (1983), 655–675
- *Recurrence relations for the coefficients in Jacobi series solutions of linear differential equations*, SIAM Journal of Mathematical Analysis 17 (1986), 1037–1052
- *A new approach to the problem of constructing recurrence relations for the Jacobi coefficients*, Zastosowania Matematyki 21 (1991), 303–326
- *Quick construction of recurrence relations for the Jacobi coefficients*, Journal of Computational and Applied Mathematics 43 (1992), 355–372
- *Recurrence relations for the coefficients of the Fourier series expansions with respect to q-classical orthogonal polynomials* (wspólnie z E. Godoyem, I. Areą, A. Ronveaux i A. Zarzo), Numerical Algorithms 23 (2000), 31–50
- *Construction of recurrences for the coefficients of expansions in q-classical orthogonal polynomials*, Journal of Computational and Applied Mathematics 153 (2003), 295–309
- *Recurrence relations for the coefficients in series expansions with respect to semi-classical orthogonal polynomials* (wspólnie z P. W.), Numerical Algorithms 35 (2004), 61–79

Funkcje hipergeometryczne

$$ {}_rF_s\left(\left.\begin{matrix} a_1, a_2, \ldots, a_r \\ b_1, b_2, \ldots, b_s \end{matrix}\right| x\right) := \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_r)_k}{(b_1)_k (b_2)_k \cdots (b_s)_k} \cdot \frac{x^k}{k!} $$

Funkcje hipergeometryczne

$$
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$$

Związki rekurencyjne minimalnego rzędu dla szczególnych funkcji hipergeometrycznych i ich bazowych odpowiedników (1985, 1997, 2000):

Funkcje hipergeometryczne

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- *Recurrence relations for hypergeometric functions of unit argument*, Mathematics of Computation 45 (1985), 521–535
- *On the differential-difference properties of the extended Jacobi polynomials*, Mathematics of Computation 44 (1985), 435–441
- *Generalized Watson's summation formula for 3F2(1)*, Journal of Computational and Applied Mathematics 86 (1997), 375–386
- *Recursion formulae for basic hypergeometric functions*, in Numerical Analysis in the 20th Century, Vol. I: Approximation Theory, eds. J. Wimp & L. Wuytack, Journal of Computational and Applied Mathematics 121 (2000), 297–312

Funkcje hipergeometryczne

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A co na to Jet Wimp: *Te wyniki są zupełnie nieoczekiwane!*

Funkcje hipergeometryczne

Dear Prof. Lewanowicz:

Thank you for your reprints; I find them very interesting, and hope to use much of the material in my book.

I am very interested in finding a recursion of minimal order for the function

$$F_n \stackrel{def}{=} {}_3F_2\left(\begin{matrix} n+a_1, \ n+a_2, \ n+a_3 \\ 2n+b_1, \ n+b_2 \end{matrix} \middle| \, 1\right).$$

Also, I suspect the recurrence, if of order < 3, could be used to establish some properties of ${}_3F_2(1)$. At any rate, it would be very surprising (and exciting) to find that F_n satisfied a recurrence of order only $\underline{\underline{3}}$!!

My best wishes

Jet Wimp

Korespondencja z J. Wimpem (1982)

Funkcje hipergeometryczne

Dear Prof. Wimp:

Thank you for your letter of September 10.

Your conjecture is true: the Saalschützian

$${}_{p+2}F_{p+1}\left(\begin{matrix}-n,\ n+\lambda,\ a_p\\ b_{p+1}\end{matrix}\middle|\,1\right), \quad \lambda=\sum_{j=1}^{p+1} b_j - \sum_{k=1}^{p} a_k - 1,$$

actually satisfies a homogeneous recurrence relation of the p^{th} order. Strictly speaking, I think that a bit more can be proved, namely we have the following

Theorem. The Saalschützian

$$f_n := {}_{p+3}F_{p+2}\left(\begin{matrix}-n,\ n+\lambda,\ a_p,\ 1\\ b_{p+2}\end{matrix}\middle|\,1\right), \quad \lambda=\sum_{j=1}^{p+2} b_j - \sum_{k=1}^{p} a_k - 2,$$

satisfies the recurrence relation

$$\sum_{m=0}^{p} I_m(n)\, f_{n-m} = \frac{(b_{p+2}-1)(n+\lambda)_n}{(n+b_{p+2}-1)(n+\lambda-p)_n},$$

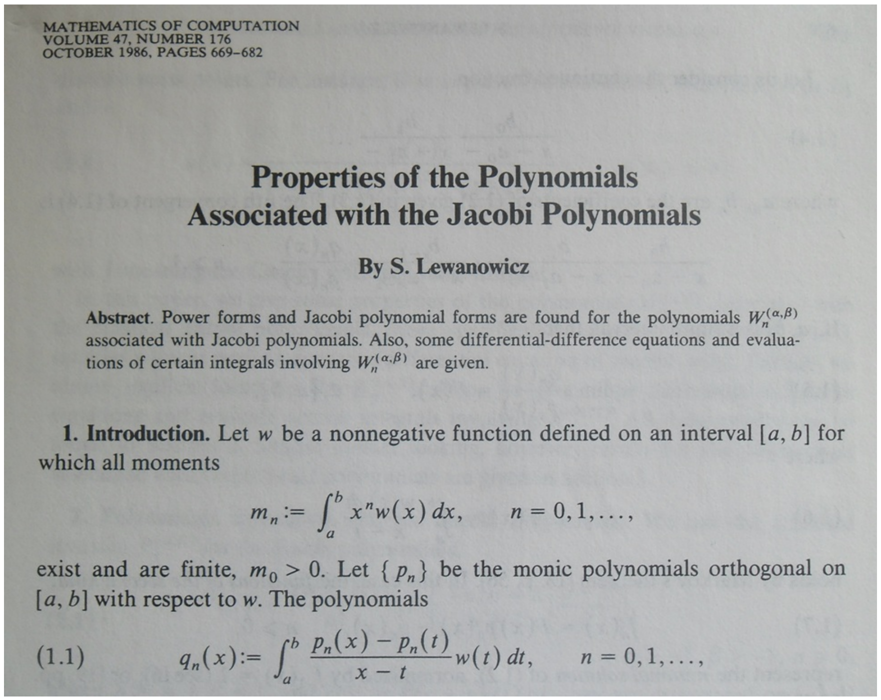
My kindest regards,

Stanisław Lewanowicz

Korespondencja z J. Wimpem (1982)

Wielomiany ortogonalne

- Własności wielomianów stowarzyszonych (1986, 1993, 1995, 1997, 2003)



MATHEMATICS OF COMPUTATION
VOLUME 47, NUMBER 176
OCTOBER 1986, PAGES 669–682

Properties of the Polynomials Associated with the Jacobi Polynomials

By S. Lewanowicz

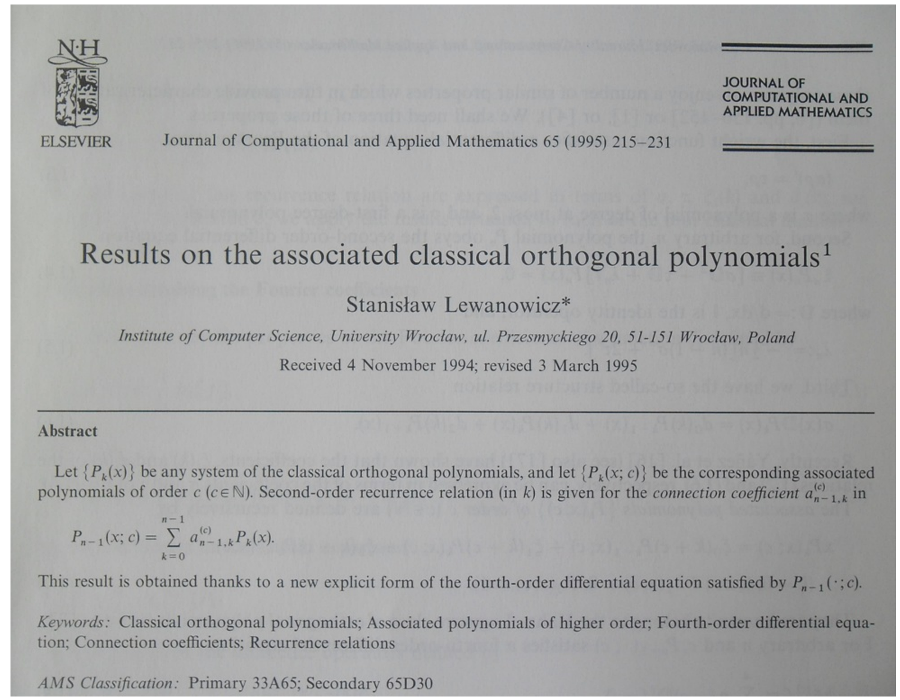
Abstract. Power forms and Jacobi polynomial forms are found for the polynomials $W_n^{(\alpha,\beta)}$ associated with Jacobi polynomials. Also, some differential-difference equations and evaluations of certain integrals involving $W_n^{(\alpha,\beta)}$ are given.

1. Introduction. Let w be a nonnegative function defined on an interval $[a, b]$ for which all moments

$$m_n := \int_a^b x^n w(x)\,dx, \qquad n = 0, 1, \dots,$$

exist and are finite, $m_0 > 0$. Let $\{p_n\}$ be the monic polynomials orthogonal on $[a, b]$ with respect to w. The polynomials

(1.1) $$q_n(x) := \int_a^b \frac{p_n(x) - p_n(t)}{x - t} w(t)\,dt, \qquad n = 0, 1, \dots,$$



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Journal of Computational and Applied Mathematics 65 (1995) 215–231
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

Results on the associated classical orthogonal polynomials[1]

Stanisław Lewanowicz*
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Received 4 November 1994; revised 3 March 1995

Abstract

Let $\{P_k(x)\}$ be any system of the classical orthogonal polynomials, and let $\{P_k(x; c)\}$ be the corresponding associated polynomials of order c ($c \in \mathbb{N}$). Second-order recurrence relation (in k) is given for the *connection coefficient* $a_{n-1,k}^{(c)}$ in

$$P_{n-1}(x; c) = \sum_{k=0}^{n-1} a_{n-1,k}^{(c)} P_k(x).$$

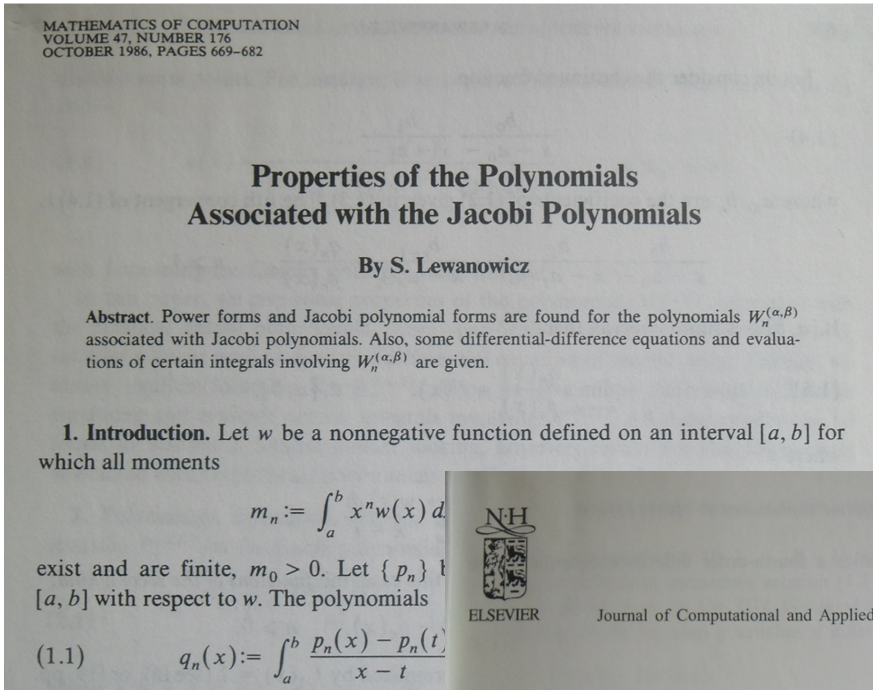
This result is obtained thanks to a new explicit form of the fourth-order differential equation satisfied by $P_{n-1}(\cdot; c)$.

Keywords: Classical orthogonal polynomials; Associated polynomials of higher order; Fourth-order differential equation; Connection coefficients; Recurrence relations

AMS Classification: Primary 33A65; Secondary 65D30

Wielomiany ortogonalne

- Własności wielomianów stowarzyszonych (1986, 1993, 1995, 1997, 2003)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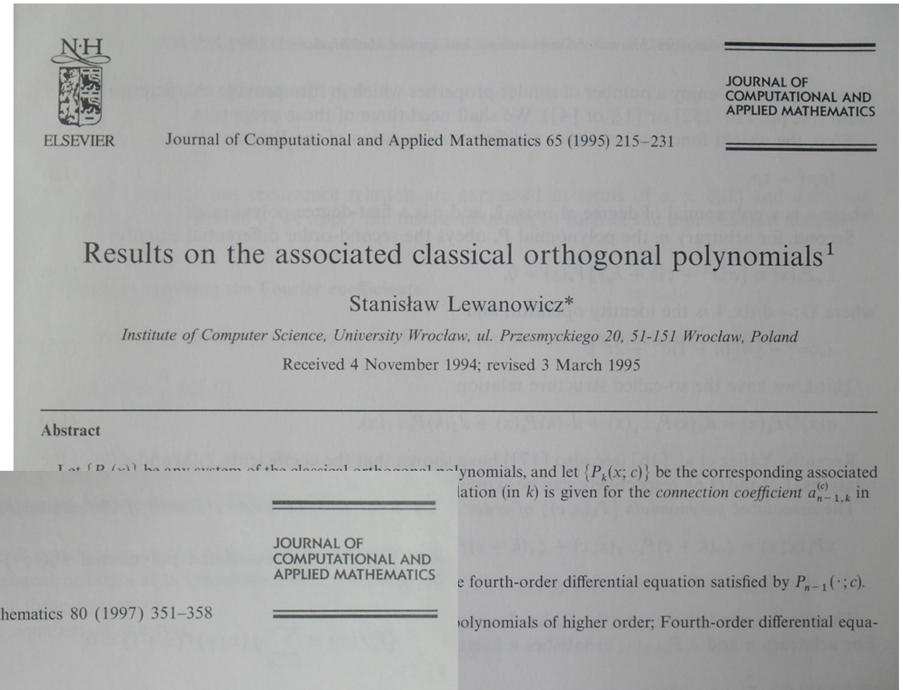
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JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

Results on the associated classical orthogonal polynomials[1]

Stanisław Lewanowicz*

Institute of Computer Science, University Wrocław, ul. Przesmyckiego 20, 51-151 Wrocław, Poland

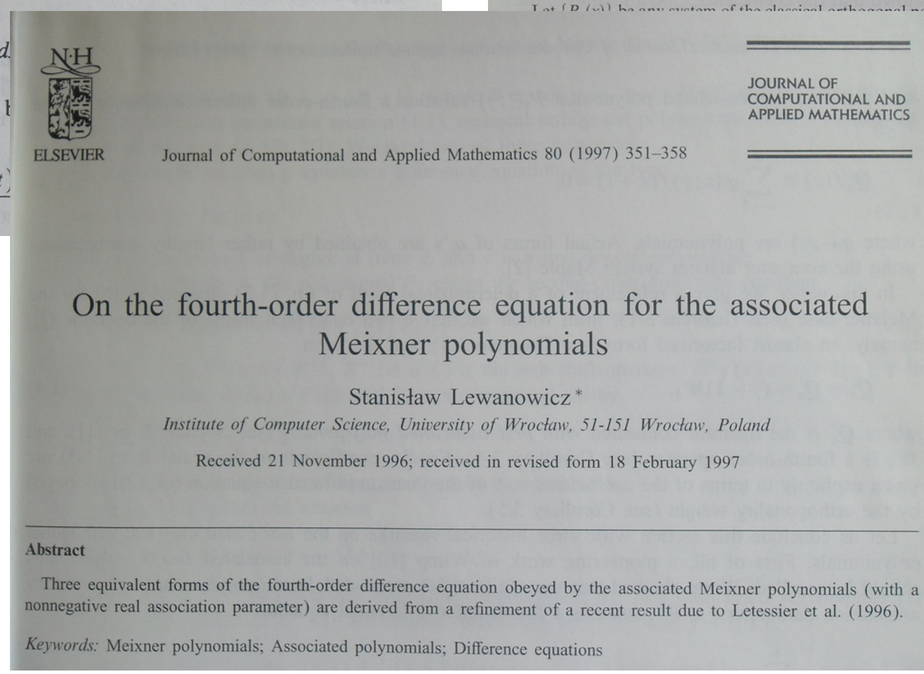
Received 4 November 1994; revised 3 March 1995

Abstract

…ynomials, and let $\{P_k(x;c)\}$ be the corresponding associated
…lation (in k) is given for the *connection coefficient* $a_{n-1,k}^{(c)}$ in

…e fourth-order differential equation satisfied by $P_{n-1}(\cdot\,;c)$.

…olynomials of higher order; Fourth-order differential equa-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Journal of Computational and Applied Mathematics 80 (1997) 351–358

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

On the fourth-order difference equation for the associated Meixner polynomials

Stanisław Lewanowicz*

Institute of Computer Science, University of Wrocław, 51-151 Wrocław, Poland

Received 21 November 1996; received in revised form 18 February 1997

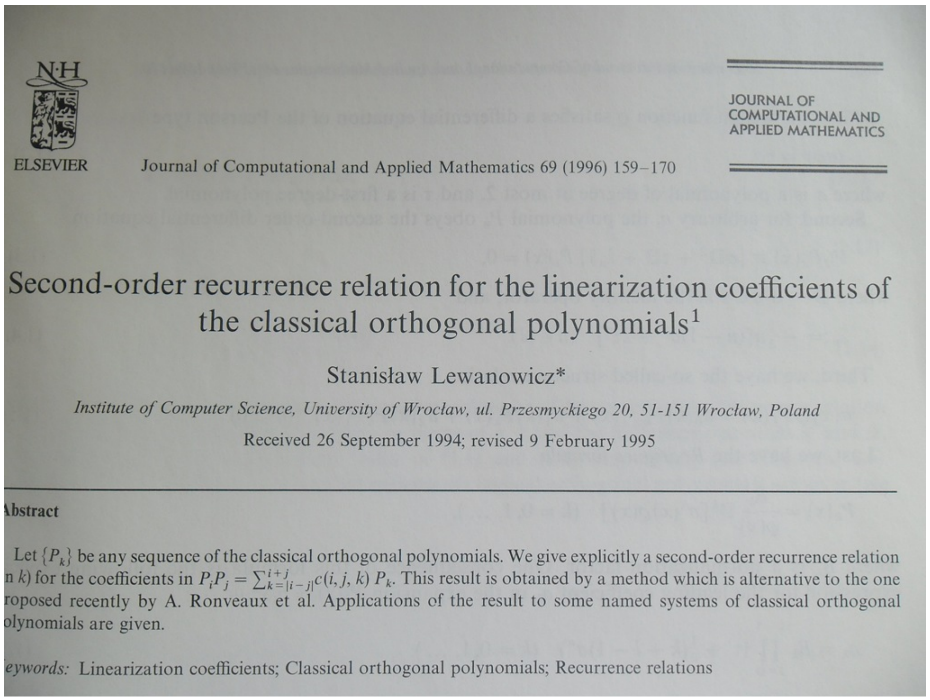
Abstract

Three equivalent forms of the fourth-order difference equation obeyed by the associated Meixner polynomials (with a nonnegative real association parameter) are derived from a refinement of a recent result due to Letessier et al. (1996).

Keywords: Meixner polynomials; Associated polynomials; Difference equations

Wielomiany ortogonalne

- Własności wielomianów stowarzyszonych (1986, 1993, 1995, 1997, 2003)
- Zagadnienia linearyzacji i koneksji (1996–1998, 2002)



JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

ELSEVIER Journal of Computational and Applied Mathematics 69 (1996) 159–170

Second-order recurrence relation for the linearization coefficients of the classical orthogonal polynomials[1]

Stanisław Lewanowicz*

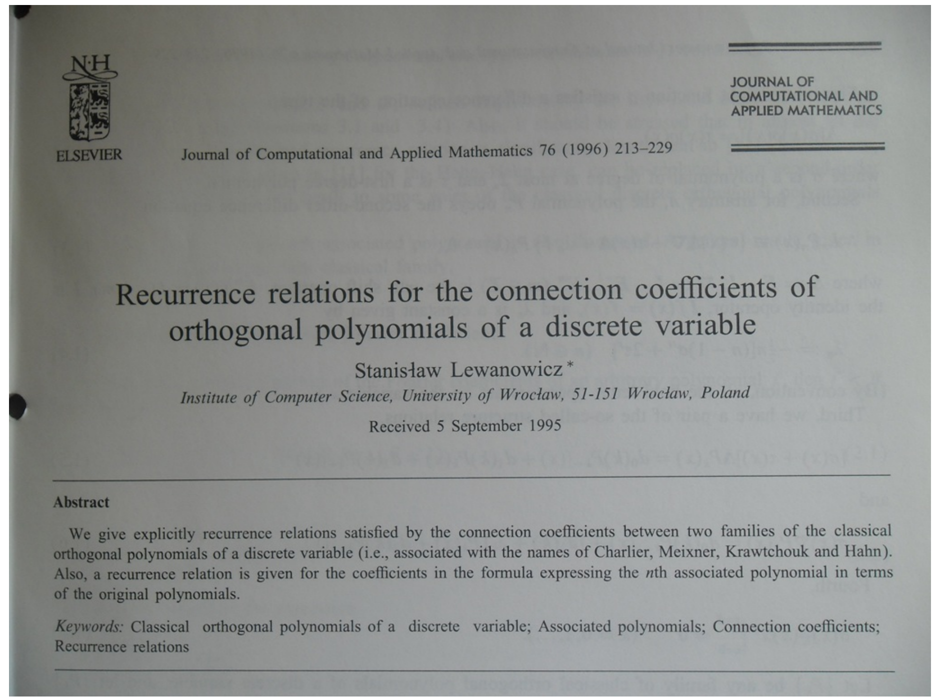
Institute of Computer Science, University of Wrocław, ul. Przesmyckiego 20, 51-151 Wrocław, Poland

Received 26 September 1994; revised 9 February 1995

bstract

Let $\{P_k\}$ be any sequence of the classical orthogonal polynomials. We give explicitly a second-order recurrence relation n k) for the coefficients in $P_iP_j = \sum_{k=|i-j|}^{i+j} c(i,j,k)\, P_k$. This result is obtained by a method which is alternative to the one roposed recently by A. Ronveaux et al. Applications of the result to some named systems of classical orthogonal olynomials are given.

eywords: Linearization coefficients; Classical orthogonal polynomials; Recurrence relations



JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

ELSEVIER Journal of Computational and Applied Mathematics 76 (1996) 213–229

Recurrence relations for the connection coefficients of orthogonal polynomials of a discrete variable

Stanisław Lewanowicz*

Institute of Computer Science, University of Wrocław, 51-151 Wrocław, Poland

Received 5 September 1995

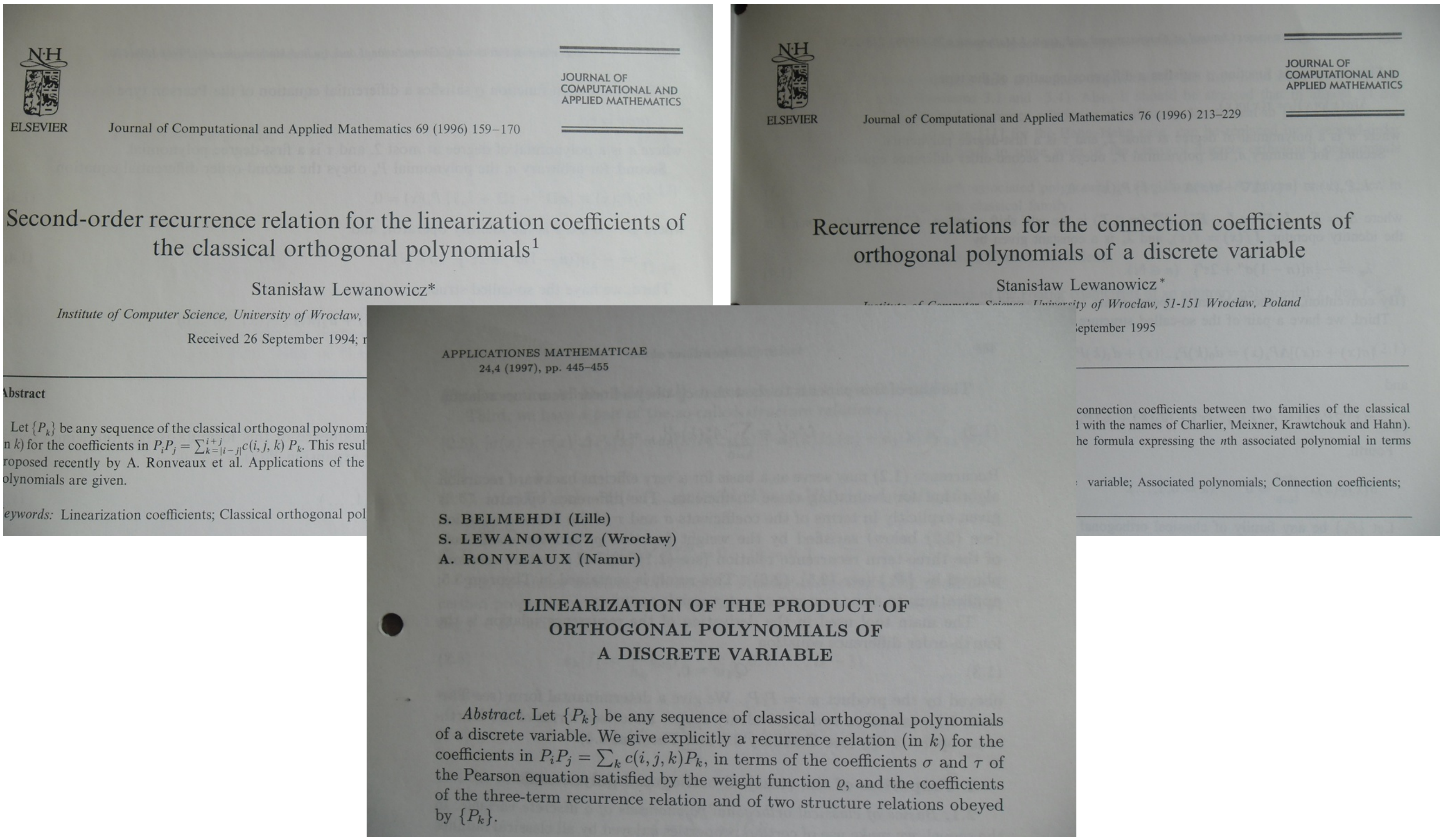
Abstract

We give explicitly recurrence relations satisfied by the connection coefficients between two families of the classical orthogonal polynomials of a discrete variable (i.e., associated with the names of Charlier, Meixner, Krawtchouk and Hahn). Also, a recurrence relation is given for the coefficients in the formula expressing the nth associated polynomial in terms of the original polynomials.

Keywords: Classical orthogonal polynomials of a discrete variable; Associated polynomials; Connection coefficients; Recurrence relations

Wielomiany ortogonalne

- Własności wielomianów stowarzyszonych (1986, 1993, 1995, 1997, 2003)
- Zagadnienia linearyzacji i koneksji (1996–1998, 2002)


JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

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…variable; Associated polynomials; Connection coefficients;

APPLICATIONES MATHEMATICAE

24,4 (1997), pp. 445–455

S. BELMEHDI (Lille)

S. LEWANOWICZ (Wrocław)

A. RONVEAUX (Namur)

LINEARIZATION OF THE PRODUCT OF ORTHOGONAL POLYNOMIALS OF A DISCRETE VARIABLE

Abstract. Let $\{P_k\}$ be any sequence of classical orthogonal polynomials of a discrete variable. We give explicitly a recurrence relation (in k) for the coefficients in $P_iP_j = \sum_k c(i,j,k)P_k$, in terms of the coefficients σ and τ of the Pearson equation satisfied by the weight function ϱ, and the coefficients of the three-term recurrence relation and of two structure relations obeyed by $\{P_k\}$.

Wielomiany ortogonalne

- Własności wielomianów stowarzyszonych (1986, 1993, 1995, 1997, 2003)
- Zagadnienia linearyzacji i koneksji (1996, 1998, 2002)
- Postać Béziera wielomianów Jacobiego na trójkącie (2006)

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Journal of Computational and Applied Mathematics 197 (2006) 520–533

JOURNAL OF
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Connections between two-variable Bernstein and Jacobi polynomials on the triangle

Stanisław Lewanowicz*, Paweł Woźny

Institute of Computer Science, University of Wrocław, ul. Przesmyckiego 20, 51-151 Wrocław, Poland

Received 7 September 2005; received in revised form 7 November 2005

Abstract

Connection coefficients between the two-variable Bernstein and Jacobi polynomial families on the triangle are given explicitly as evaluations of two-variable Hahn polynomials. Dual two-variable Bernstein polynomials are introduced. Explicit formula in terms of two-variable Jacobi polynomials and a recurrence relation are given.

Keywords: Two-variable Bernstein polynomials; Two-variable Jacobi polynomials; Two-variable Hahn polynomials; Dual two-variable Bernstein polynomials; Connection relations

Wielomiany ortogonalne

- Własności wielomianów stowarzyszonych (1986, 1993, 1995, 1997, 2003)
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$\Rightarrow$ Zastosowania w grafice

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- Własności wielomianów stowarzyszonych (1986, 1993, 1995, 1997, 2003)
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三角曲面显式最佳降多阶的一个新颖算法

胡倩倩 王国瑾*

(浙江大学计算机图象图形研究所; 浙江大学 CAD&CG 国家重点实验室, 杭州 310027)

摘要 计算机辅助设计(CAD)系统中的数据通讯和数据压缩经常需要把参数曲面近似地降阶. 而其中对三角曲面一次性降多阶是一个悬而未决的技术难题. 文中把三角 Jacobi 基正交的代数性质应用到几何逼近, 借助三角 Bernstein 基和三角 Jacobi 基相互转换的最新成果, 自然地诱导出三角 Bézier 曲面一次性降多阶的一个新颖算法. 此算法具有误差预测、显式表达、机时最少、精度最佳的 4 个特点: 第一, 降阶前可迅速判断是否存在满足给定公差的降多阶曲面; 第二, 全部降多阶运算仅需对曲面的控制顶点序列按词典顺序排序所写成的列向量执行一个矩阵乘法; 第三, 此矩阵无需临时计算而是从数据库中直接调用; 第四, 这张降多阶曲面在 L_2 范数意义下达到最佳逼近效果. 数值实验证实了理论推导的正确性, 表明此算法对 CAD 系统的产品信息处理将会带来显著的应用效益.

关键词 计算机辅助设计 数据压缩 三角 Bézier 曲面 降多阶 Bernstein 多项式 Jacobi 多项式 L_2 范数

Wielomiany ortogonalne

- Własności wielomianów stowarzyszonych (1986, 1993, 1995, 1997, 2003)
- Zagadnienia linearyzacji i koneksji (1996, 1998, 2002)
- Postać Béziera wielomianów Jacobiego na trójkącie (2006)

其中

$$\varphi_{k,l}(n,i,j) = \frac{(-1)^{k-l}\, 2\cdot n!(i-n)_l (n-k+1)_{k-l}}{(n+k+2)!}\sqrt{\frac{(2l+1)(k+1)(k+l+1)!}{l!(l+1)_{k+1}}} \times Q_l(j;0,0,n-i)Q_{k-l}(i;0,2l+1,n-l), \qquad (7)$$

且

$$Q_m(t;\mu,\nu,M) = {}_3F_2\left(\begin{matrix}-m, m+\mu+\nu+1, -t\\ \mu+1, -M\end{matrix}\middle|\, 1\right).$$

证明 将 $\alpha=\beta=\gamma=1/2$ 代入 Lewanowicz 等[16]文的定理 2.1 即得证.

引理 2 对任意 $k,l,n\in\mathbb{N}_0$ 使得 $l\leqslant k\leqslant n$，则双变量 Jacobi 基 $J_{k,l}$ 可由双变量 Bernstein 基 $B^n_{i,j}$ 表示:

$$J_{k,l}(s,t) = \sum_{i+j\leqslant n} \zeta_{i,j}(n,k,l)B^n_{i,j}(s,t), \qquad (8)$$

其中

$$\zeta_{k,l}(n,i,j) = \frac{(-1)^l (i-n)_l (n-k+1)_{k-l}}{(-n)_k}\sqrt{\frac{(2l+1)(k+1)(k+l+1)!}{l!(l+1)_{k+1}}} \times Q_l(j;0,0,n-i)Q_{k-l}(i;0,2l+1,n-l), \qquad (9)$$

且 $Q_m(t;\mu,\nu,M)$ 如引理 1 所示.

证明 将 $\alpha=\beta=\gamma=1/2$ 代入 Lewanowicz 等文[16]的定理 2.3 即得证.

Operatory rzutowe

$\mathcal{Q} : C[-1, 1] \to \Pi_n$

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$\mathcal{Q} : C[-1, 1] \to \Pi_n, \quad \mathcal{Q}(w) = w \quad (w \in \Pi_n)$

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$$\mathcal{Q} : C[-1,1] \to \Pi_n, \quad \mathcal{Q}(w) = w \quad (w \in \Pi_n), \quad \|f - \mathcal{Q}f\|_\infty \le (1 + \|\mathcal{Q}\|) \min_{w \in \Pi_n} \|f - w\|_\infty$$

Operatory rzutowe

$\mathcal{Q} : C[-1,1] \to \Pi_n, \quad \mathcal{Q}(w) = w \quad (w \in \Pi_n), \quad \|f - \mathcal{Q}f\|_\infty \le (1 + \|\mathcal{Q}\|) \min_{w \in \Pi_n} \|f - w\|_\infty, \quad \|\mathcal{Q}\|$ – mała!

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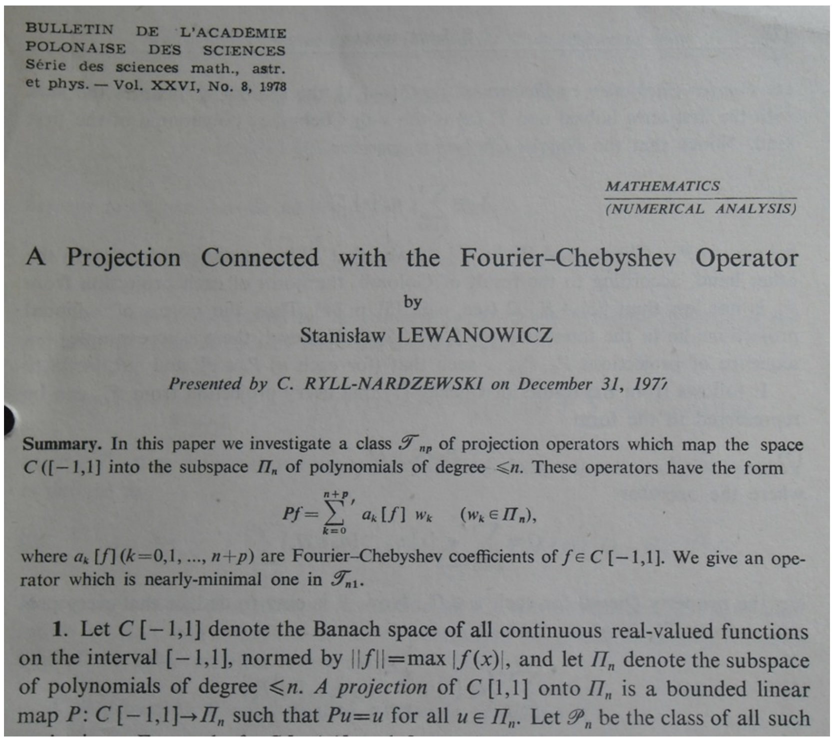
- Własności operatorów rzutowych. Operatory Lewanowicza (1978, 1979, 1982, 1993)



BULLETIN DE L'ACADEMIE POLONAISE DES SCIENCES
Série des sciences math., astr. et phys. — Vol. XXVI, No. 8, 1978

MATHEMATICS (NUMERICAL ANALYSIS)

A Projection Connected with the Fourier–Chebyshev Operator

by
Stanisław LEWANOWICZ

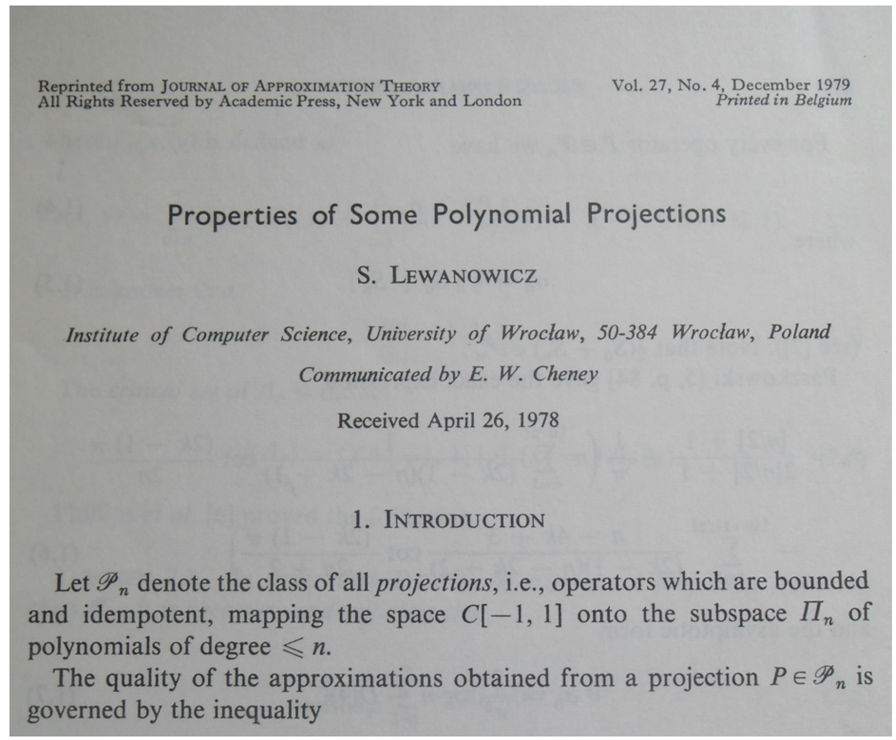
Presented by C. RYLL-NARDZEWSKI on December 31, 1977

Summary. In this paper we investigate a class $\mathcal{T}_{np}$ of projection operators which map the space $C([-1,1]$ into the subspace Π_n of polynomials of degree $\leqslant n$. These operators have the form

$$Pf = \sum_{k=0}^{n+p}{}' a_k[f]\, w_k \qquad (w_k \in \Pi_n),$$

where $a_k[f]$ $(k=0,1,\ldots,n+p)$ are Fourier–Chebyshev coefficients of $f \in C[-1,1]$. We give an operator which is nearly-minimal one in $\mathcal{T}_{n1}$.

1. Let $C[-1,1]$ denote the Banach space of all continuous real-valued functions on the interval $[-1,1]$, normed by $\|f\| = \max |f(x)|$, and let Π_n denote the subspace of polynomials of degree $\leqslant n$. *A projection* of $C[1,1]$ onto Π_n is a bounded linear map $P: C[-1,1] \to \Pi_n$ such that $Pu = u$ for all $u \in \Pi_n$. Let $\mathcal{P}_n$ be the class of all such



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 Printed in Belgium

Properties of Some Polynomial Projections

S. LEWANOWICZ

Institute of Computer Science, University of Wrocław, 50-384 Wrocław, Poland

Communicated by E. W. Cheney

Received April 26, 1978

1. INTRODUCTION

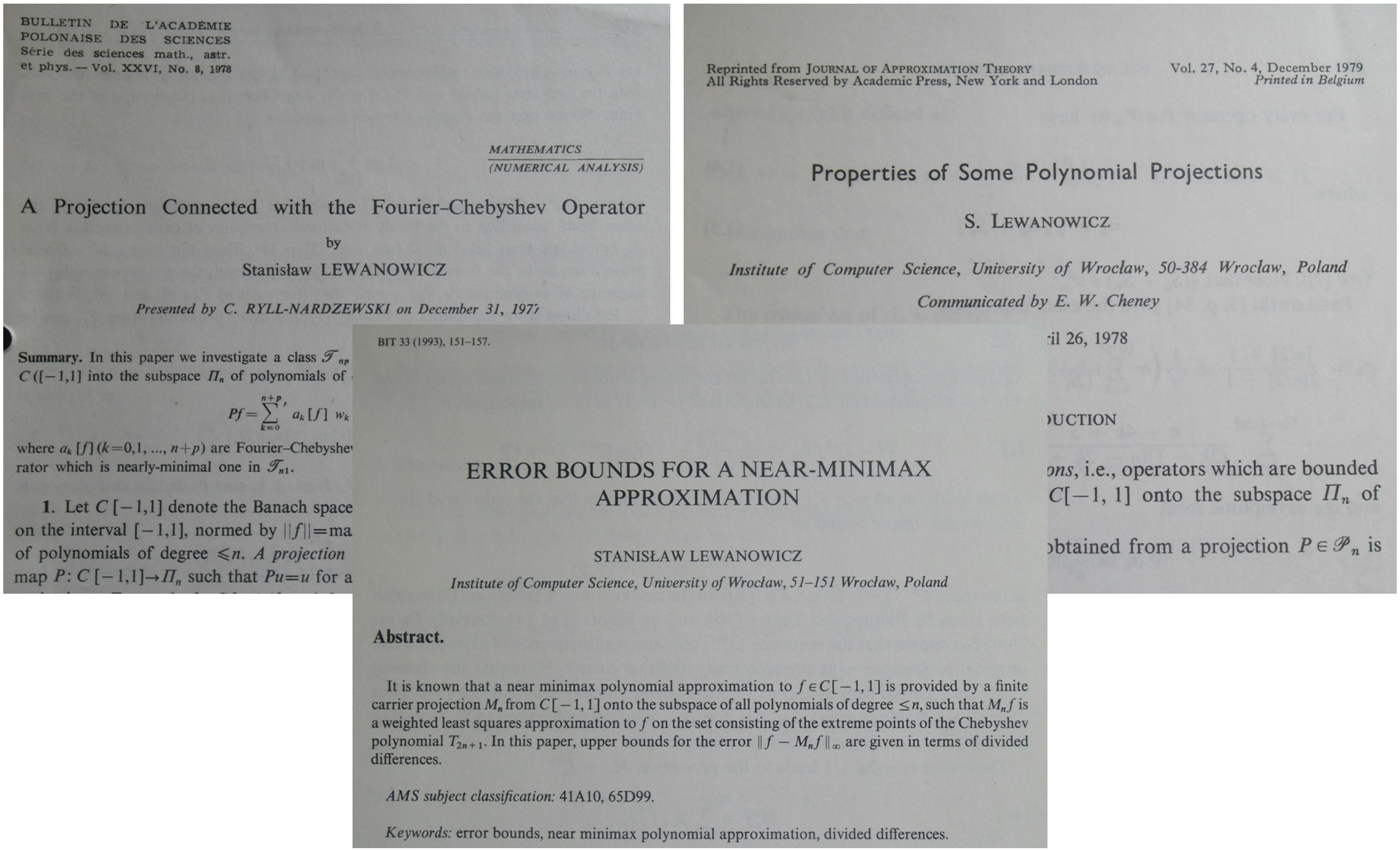
Let $\mathcal{P}_n$ denote the class of all *projections*, i.e., operators which are bounded and idempotent, mapping the space $C[-1,1]$ onto the subspace Π_n of polynomials of degree $\leqslant n$.

The quality of the approximations obtained from a projection $P \in \mathcal{P}_n$ is governed by the inequality

Operatory rzutowe

$\mathcal{Q} : C[-1,1] \to \Pi_n, \quad \mathcal{Q}(w) = w \quad (w \in \Pi_n), \quad \|f - \mathcal{Q}f\|_\infty \le (1 + \|\mathcal{Q}\|) \min_{w \in \Pi_n} \|f - w\|_\infty, \quad \|\mathcal{Q}\|$ – mała!

- Własności operatorów rzutowych. Operatory Lewanowicza (1978, 1979, 1982, 1993)



BULLETIN DE L'ACADÉMIE POLONAISE DES SCIENCES
Série des sciences math., astr. et phys. — Vol. XXVI, No. 8, 1978

MATHEMATICS
(NUMERICAL ANALYSIS)

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BIT 33 (1993), 151–157.

ERROR BOUNDS FOR A NEAR-MINIMAX APPROXIMATION

STANISŁAW LEWANOWICZ

Institute of Computer Science, University of Wrocław, 51-151 Wrocław, Poland

Abstract.

It is known that a near minimax polynomial approximation to $f \in C[-1,1]$ is provided by a finite carrier projection M_n from $C[-1,1]$ onto the subspace of all polynomials of degree $\leq n$, such that $M_n f$ is a weighted least squares approximation to f on the set consisting of the extreme points of the Chebyshev polynomial T_{2n+1}. In this paper, upper bounds for the error $\|f - M_n f\|_\infty$ are given in terms of divided differences.

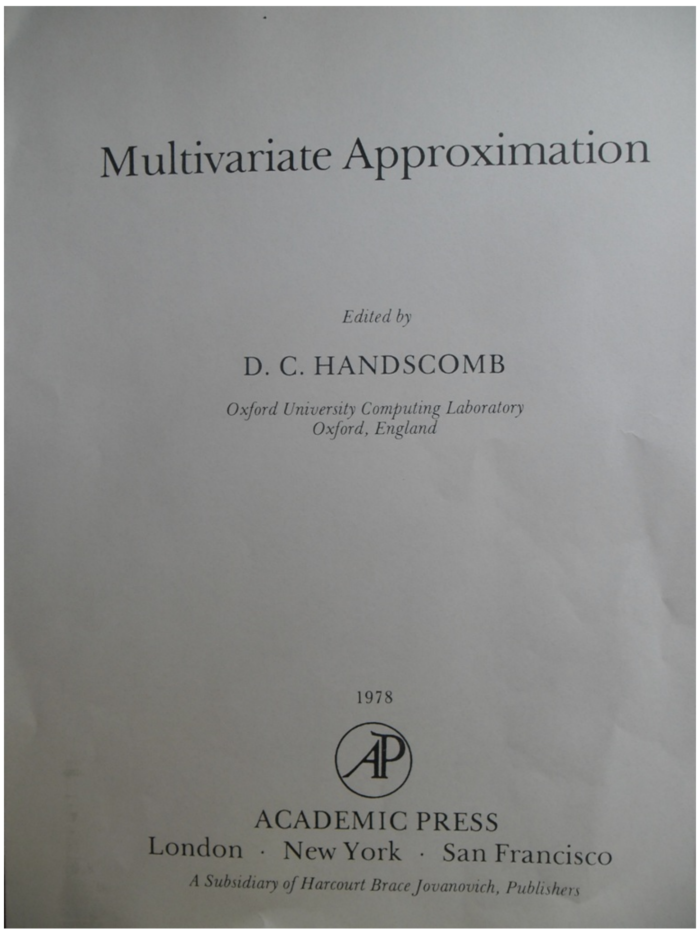
AMS subject classification: 41A10, 65D99.

Keywords: error bounds, near minimax polynomial approximation, divided differences.

Operatory rzutowe

$\mathcal{Q} : C[-1,1] \to \Pi_n, \quad \mathcal{Q}(w) = w \quad (w \in \Pi_n), \quad \|f - \mathcal{Q}f\|_\infty \le (1 + \|\mathcal{Q}\|) \min_{w \in \Pi_n} \|f - w\|_\infty, \quad \|\mathcal{Q}\|$ – mała!

- Własności operatorów rzutowych. Operatory Lewanowicza (1978, 1979, 1982, 1993)



Multivariate Approximation

Edited by

D. C. HANDSCOMB

Oxford University Computing Laboratory
Oxford, England

1978

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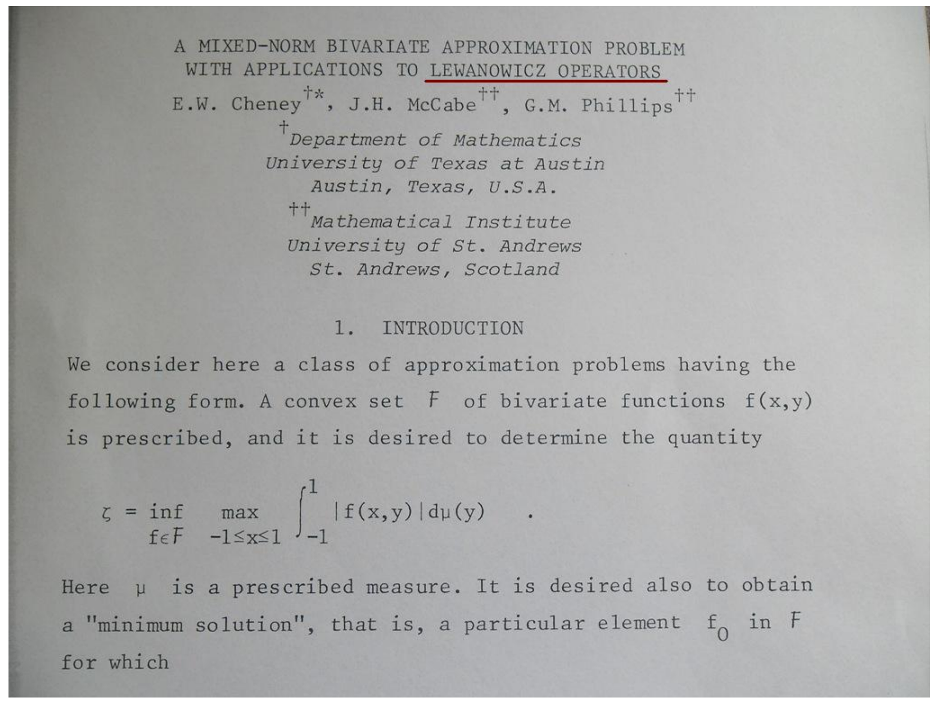

A MIXED-NORM BIVARIATE APPROXIMATION PROBLEM
WITH APPLICATIONS TO LEWANOWICZ OPERATORS

E.W. Cheney[†*], J.H. McCabe[††], G.M. Phillips[††]

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University of Texas at Austin
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University of St. Andrews
St. Andrews, Scotland

1. INTRODUCTION

We consider here a class of approximation problems having the following form. A convex set F of bivariate functions $f(x,y)$ is prescribed, and it is desired to determine the quantity

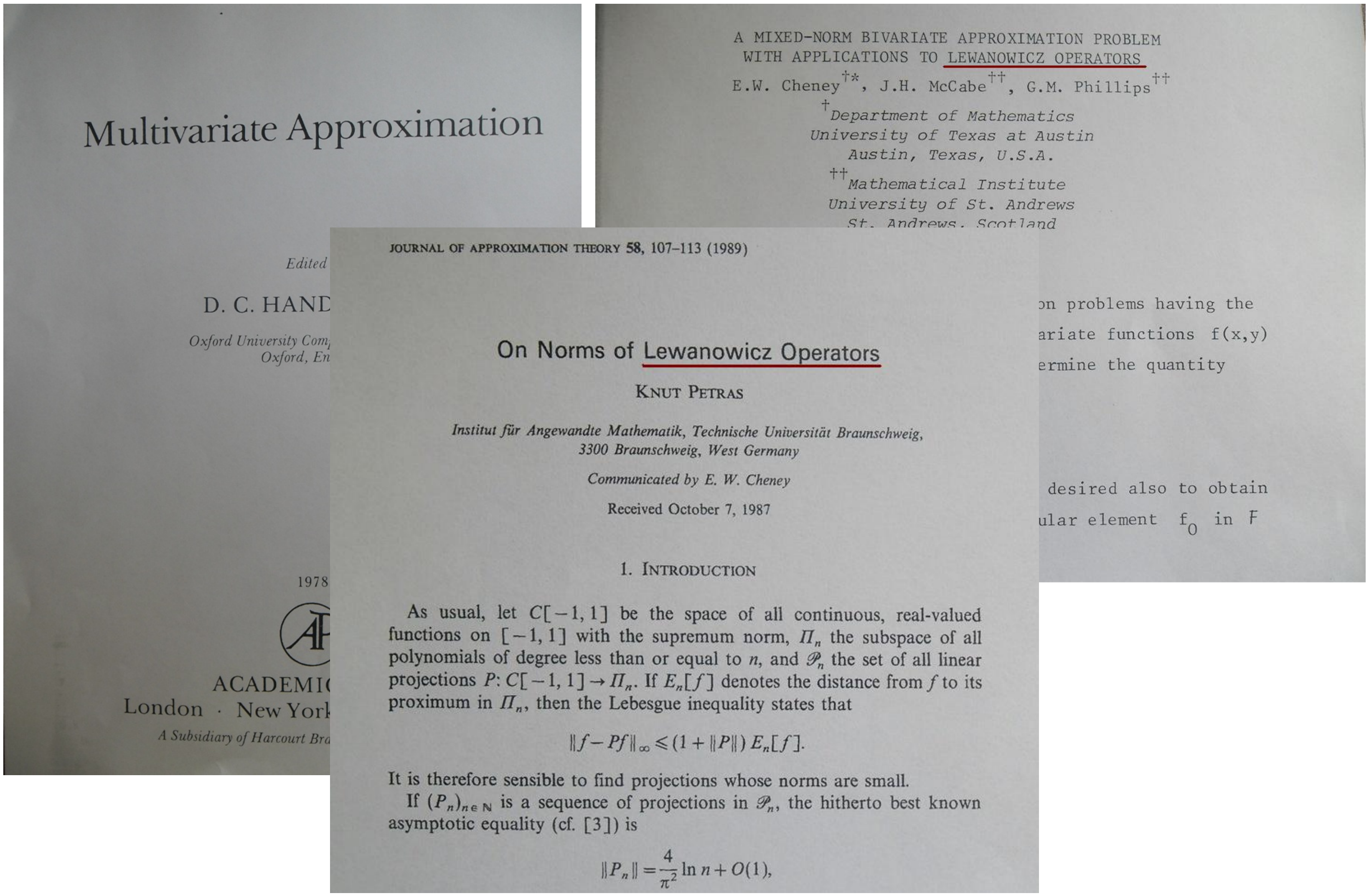
$$\zeta = \inf_{f \in F} \max_{-1 \le x \le 1} \int_{-1}^{1} |f(x,y)|\, d\mu(y) \quad .$$

Here μ is a prescribed measure. It is desired also to obtain a "minimum solution", that is, a particular element f_0 in F for which

Operatory rzutowe

$\mathcal{Q} : C[-1,1] \to \Pi_n, \quad \mathcal{Q}(w) = w \quad (w \in \Pi_n), \quad \|f - \mathcal{Q}f\|_\infty \le (1 + \|\mathcal{Q}\|) \min_{w \in \Pi_n} \|f - w\|_\infty, \quad \|\mathcal{Q}\|$ – mała!

- Własności operatorów rzutowych. Operatory Lewanowicza (1978, 1979, 1982, 1993)



Multivariate Approximation

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A MIXED-NORM BIVARIATE APPROXIMATION PROBLEM
WITH APPLICATIONS TO LEWANOWICZ OPERATORS

E.W. Cheney[†*], J.H. McCabe[††], G.M. Phillips[††]

[†]Department of Mathematics
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[††]Mathematical Institute
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St. Andrews, Scotland

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JOURNAL OF APPROXIMATION THEORY 58, 107–113 (1989)

On Norms of Lewanowicz Operators

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Communicated by E. W. Cheney

Received October 7, 1987

1. INTRODUCTION

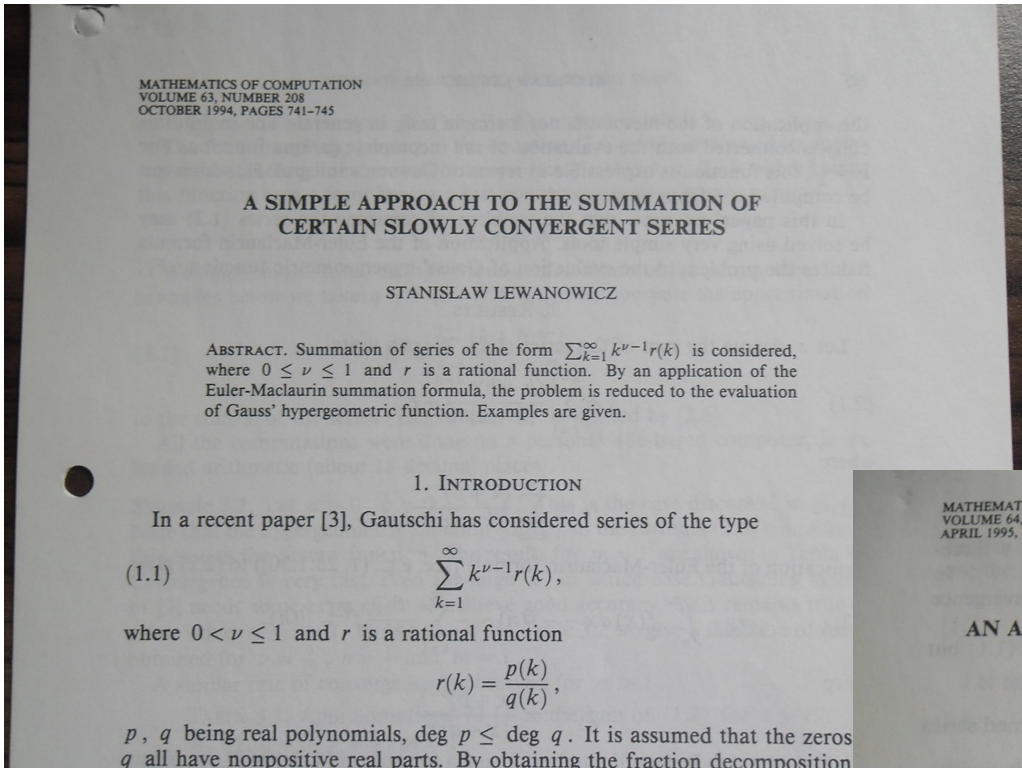
As usual, let $C[-1,1]$ be the space of all continuous, real-valued functions on $[-1,1]$ with the supremum norm, Π_n the subspace of all polynomials of degree less than or equal to n, and $\mathscr{P}_n$ the set of all linear projections $P: C[-1,1] \to \Pi_n$. If $E_n[f]$ denotes the distance from f to its proximum in Π_n, then the Lebesgue inequality states that

$$\|f - Pf\|_\infty \leqslant (1 + \|P\|)\, E_n[f].$$

It is therefore sensible to find projections whose norms are small.
If $(P_n)_{n \in \mathbb{N}}$ is a sequence of projections in $\mathscr{P}_n$, the hitherto best known asymptotic equality (cf. [3]) is

$$\|P_n\| = \frac{4}{\pi^2} \ln n + O(1),$$

Przyspieszanie zbieżności szeregów



MATHEMATICS OF COMPUTATION
VOLUME 63, NUMBER 208
OCTOBER 1994, PAGES 741–745

A SIMPLE APPROACH TO THE SUMMATION OF CERTAIN SLOWLY CONVERGENT SERIES

STANISLAW LEWANOWICZ

ABSTRACT. Summation of series of the form $\sum_{k=1}^{\infty} k^{\nu-1} r(k)$ is considered, where $0 \le \nu \le 1$ and r is a rational function. By an application of the Euler-Maclaurin summation formula, the problem is reduced to the evaluation of Gauss' hypergeometric function. Examples are given.

1. INTRODUCTION

In a recent paper [3], Gautschi has considered series of the type

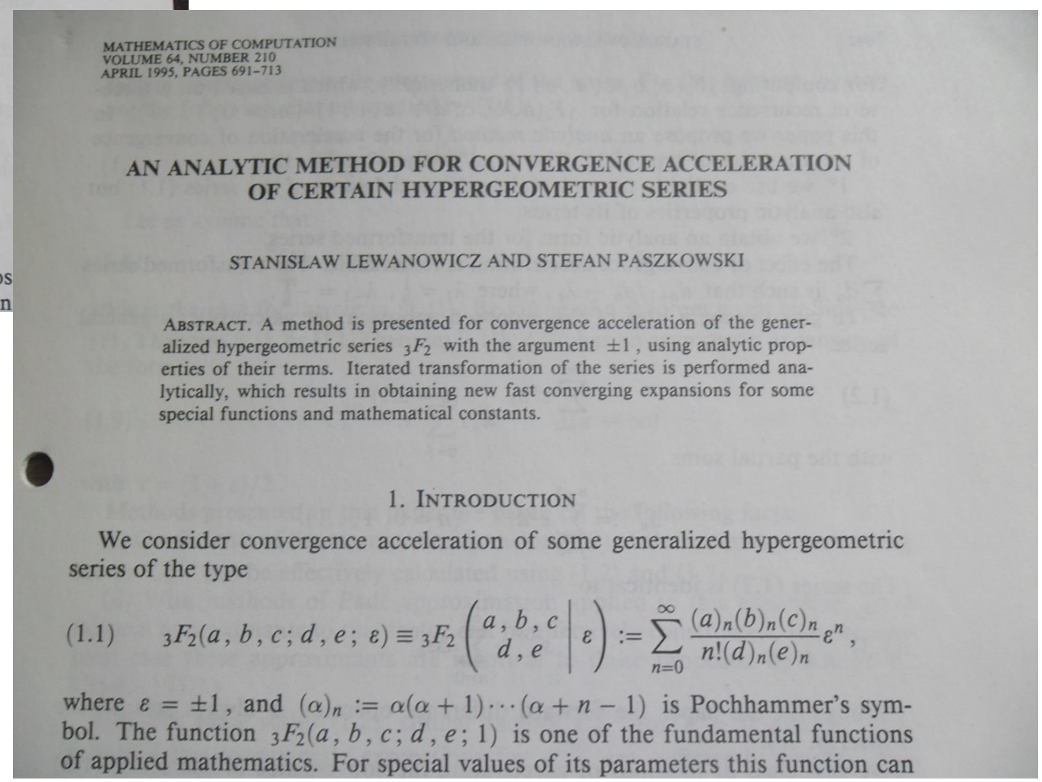
$$\sum_{k=1}^{\infty} k^{\nu-1} r(k)\,, \qquad (1.1)$$

where $0 < \nu \le 1$ and r is a rational function

$$r(k) = \frac{p(k)}{q(k)}\,,$$

p, q being real polynomials, deg $p \le$ deg q. It is assumed that the zeros q all have nonpositive real parts. By obtaining the fraction decomposition

Bardzo prosta, ale skuteczna metoda (1994)



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AN ANALYTIC METHOD FOR CONVERGENCE ACCELERATION OF CERTAIN HYPERGEOMETRIC SERIES

STANISLAW LEWANOWICZ AND STEFAN PASZKOWSKI

ABSTRACT. A method is presented for convergence acceleration of the generalized hypergeometric series ${}_3F_2$ with the argument ± 1, using analytic properties of their terms. Iterated transformation of the series is performed analytically, which results in obtaining new fast converging expansions for some special functions and mathematical constants.

1. INTRODUCTION

We consider convergence acceleration of some generalized hypergeometric series of the type

$$ {}_3F_2(a, b, c; d, e; \varepsilon) \equiv {}_3F_2\left(\begin{matrix} a, b, c \\ d, e \end{matrix} \middle| \, \varepsilon\right) := \sum_{n=0}^{\infty} \frac{(a)_n (b)_n (c)_n}{n! (d)_n (e)_n} \varepsilon^n\,, \qquad (1.1)$$

where $\varepsilon = \pm 1$, and $(\alpha)_n := \alpha(\alpha+1)\cdots(\alpha+n-1)$ is Pochhammer's symbol. The function ${}_3F_2(a, b, c; d, e; 1)$ is one of the fundamental functions of applied mathematics. For special values of its parameters this function can

Przekształcenia analityczne $\Rightarrow$ nowe szybkozbieżne rozwinięcia (1995)

Nowe rodziny wielomianów

- Uogólnione wielomiany Bernsteina jednej i wielu zmiennych (2004, 2008)

BIT Numerical Mathematics **44:** 63–78, 2004.

63

GENERALIZED BERNSTEIN POLYNOMIALS *

STANISŁAW LEWANOWICZ[1] and PAWEŁ WOŹNY[1]

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Abstract.

We introduce polynomials $B_i^n(x;\omega|q)$, depending on two parameters q and ω, which generalize classical Bernstein polynomials, discrete Bernstein polynomials defined by Sablonnière, as well as q-Bernstein polynomials introduced by Phillips. Basic properties of the new polynomials are given. Also, formulas relating $B_i^n(x;\omega|q)$, big q-Jacobi and q-Hahn (or dual q-Hahn) polynomials are presented.

AMS subject classification (2000): 41A10, 65D17, 33D45.

Key words: generalized Bernstein polynomials, generalized Bézier form, generalized de Casteljau algorithm, degree elevation, big q-Jacobi polynomials, q-Hahn polynomials, dual q-Hahn polynomials, orthogonal polynomial expansion.

1 Introduction.

We define *generalized Bernstein polynomials of degree* n ($n \in \mathbb{N}$) by

$$B_i^n(x;\omega|q) := \frac{1}{(\omega;q)_n}\begin{bmatrix} n \\ i \end{bmatrix}_q x^i(\omega x^{-1};q)_i(x;q)_{n-i} \quad (i = 0, 1, \ldots, n), \tag{1.1}$$

Numer Algor (2008) 49:199–220
DOI 10.1007/s11075-008-9168-9

ORIGINAL PAPER

Multivariate generalized Bernstein polynomials: identities for orthogonal polynomials of two variables

Stanisław Lewanowicz · Paweł Woźny · Iván Area · Eduardo Godoy

Received: 20 June 2007 / Accepted: 4 January 2008 / Published online: 21 February 2008

Abstract We introduce polynomials $B_k^n(x;\omega|q)$ of total degree n, where $k = (k_1, \ldots, k_d) \in \mathbb{N}_0^d$, $0 \le k_1 + \ldots + k_d \le n$, and $x = (x_1, x_2, \ldots, x_d) \in \mathbb{R}^d$, depending on two parameters q and ω, which generalize the multivariate classical and discrete Bernstein polynomials. For $\omega = 0$, we obtain an extension of univariate q-Bernstein polynomials, introduced by Phillips (Ann Numer Math 4:511–518, 1997). Basic properties of the new polynomials are given, including recurrence relations, q-differentiation rules and de Casteljau algorithm. For the case $d = 2$, connections between $B_k^n(x;\omega|q)$ and bivariate orthogonal big q-Jacobi polynomials—introduced recently by the first two authors—are given, with the connection coefficients being expressed in terms of bivariate q-Hahn polynomials. As limiting forms of these relations, we give connections between bivariate q-Bernstein and Dunkl's (little) q-Jacobi polynomials (SIAM J Algebr Discrete Methods 1:137–151, 1980), as well as between bivariate discrete Bernstein and Hahn polynomials.

Dedicated to the memory of Luigi Gatteschi.

Nowe rodziny wielomianów

- Uogólnione wielomiany Bernsteina jednej i wielu zmiennych (2004, 2008)

BIT Numerical Mathematics **44:** 63–78, 2004.

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$\Downarrow$

Najczęściej cytowana praca Lewanowicza

Numer Algor (2008) 49:199–220
DOI 10.1007/s11075-008-9168-9

ORIGINAL PAPER

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Abstract We introduce polynomials $B_{\boldsymbol{k}}^n(\boldsymbol{x};\omega|q)$ of total degree n, where $\boldsymbol{k} = (k_1,\ldots,k_d) \in \mathbb{N}_0^d$, $0 \le k_1 + \ldots + k_d \le n$, and $\boldsymbol{x} = (x_1, x_2,\ldots,x_d) \in \mathbb{R}^d$, depending on two parameters q and ω, which generalize the multivariate classical and discrete Bernstein polynomials. For $\omega = 0$, we obtain an extension of univariate q-Bernstein polynomials, introduced by Phillips (Ann Numer Math 4:511–518, 1997). Basic properties of the new polynomials are given, including recurrence relations, q-differentiation rules and de Casteljau algorithm. For the case $d = 2$, connections between $B_{\boldsymbol{k}}^n(\boldsymbol{x};\omega|q)$ and bivariate orthogonal big q-Jacobi polynomials—introduced recently by the first two authors—are given, with the connection coefficients being expressed in terms of bivariate q-Hahn polynomials. As limiting forms of these relations, we give connections between bivariate q-Bernstein and Dunkl's (little) q-Jacobi polynomials (SIAM J Algebr Discrete Methods 1:137–151, 1980), as well as between bivariate discrete Bernstein and Hahn polynomials.

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Nowe rodziny wielomianów

- Uogólnione wielomiany Bernsteina jednej i wielu zmiennych (2004, 2008)

$$B_k^n(x;\, \omega|q) = \frac{1}{(\omega;q)_n} \begin{bmatrix} n \\ k \end{bmatrix}_q x^k (\omega/x;q)_k (x;q)_{n-k} \qquad (0 \le k \le n)$$

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$$\xrightarrow{q \to 1}$$

$$\boxed{\text{Wielomiany Bernsteina (S.N. Bernstein, 1912)}}$$

Nowe rodziny wielomianów

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$q \to 1$

Wielomiany Bernsteina
(S.N. Bernstein, 1912)

$\omega = 0$

Wielomiany q-Bernsteina
(G.M. Phillips, 1997)

Nowe rodziny wielomianów

- Uogólnione wielomiany Bernsteina jednej i wielu zmiennych (2004, 2008)

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$q \to 1$

Wielomiany Bernsteina
(S.N. Bernstein, 1912)

$\omega = 0$

Dyskretne wielomiany Bernsteina
(M. Neamtu, 1991)

Wielomiany q-Bernsteina
(G.M. Phillips, 1997)

Nowe rodziny wielomianów

- Uogólnione wielomiany Bernsteina jednej i wielu zmiennych (2004, 2008)

$$\Downarrow$$

Uogólnione operatory Bernsteina i ich zastosowania w teorii aproksymacji

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J. Math. Anal. Appl. 340 (2008) 1096–1108

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Properties of convergence for ω, q-Bernstein polynomials ☆

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Submitted by Richard M. Aron

Abstract

In this paper, we discuss properties of the ω, q-Bernstein polynomials $B_n^{\omega,q}(f; x)$ introduced by S. Lewanowicz and P. Woźny in [S. Lewanowicz, P. Woźny, Generalized Bernstein polynomials, BIT 44 (1) (2004) 63–78], where $f \in C[0,1]$, $\omega, q > 0$, $\omega \neq 1, q^{-1}, \dots, q^{-n+1}$. When $\omega = 0$, we recover the q-Bernstein polynomials introduced by [G.M. Phillips, Bernstein polynomials based on the q-integers, Ann. Numer. Math. 4 (1997) 511–518]; when $q = 1$, we recover the classical Bernstein polynomials. We compute the second moment of $B_n^{\omega,q}(t^2; x)$, and demonstrate that if f is convex and $\omega, q \in (0,1)$ or $(1, \infty)$, then $B_n^{\omega,q}(f; x)$ are monotonically decreasing in n for all $x \in [0,1]$. We prove that for $\omega \in (0,1)$, $q_n \in (0,1]$, the sequence $\{B_n^{\omega,q_n}(f)\}_{n\geqslant 1}$ converges to f uniformly on $[0,1]$ for each $f \in C[0,1]$ if and only if $\lim_{n\to\infty} q_n = 1$. For fixed $\omega, q \in (0,1)$, we prove that the sequence $\{B_n^{\omega,q}(f)\}$ converges for each $f \in C[0,1]$ and obtain the estimates for the rate of convergence of $\{B_n^{\omega,q}(f)\}$ by the modulus of continuity of f, and the estimates are sharp in the sense of order for Lipschitz continuous functions.

Keywords: ω, q-Bernstein polynomials; Limit ω, q-Bernstein operators; Rate of convergence; Modulus of continuity

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LINEAR ALGEBRA and its Applications

Shape-preserving properties of ω, q-Bernstein polynomials ☆

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Totally positivity
Variation-diminishing

ABSTRACT

In this paper, we discuss shape-preserving properties of the ω, q-Bernstein polynomials $B_n^{\omega,q}(f; x)$ introduced by Lewanowicz and Wozny in [S. Lewanowicz, P. Wozny, Generalized Bernstein polynomials, BIT 44(1) (2004) 63–78] for $\omega, q \in (0,1)$. When $\omega = 0$, we recover the q-Bernstein polynomials introduced by Phillips [G.M. Phillips, Bernstein polynomials based on the q-integers, Ann. Numer. Math. 4 (1997) 511–518]; when $q = 1$, we recover the classical Bernstein polynomials. For $\omega, q \in (0,1)$, we show that the basic ω, q-Bernstein polynomial basis is a normalized totally positive basis on [0,1] and that the ω, q-Bernstein operators $B_n^{\omega,q}$ on $C[0,1]$ are variation-diminishing, monotonicity-preserving and convexity-preserving. We also show that the ω, q-Bernstein polynomials of a convex function f in the case $\omega, q \in (0,1)$ are monotonic in the parameters ω and q, for fixed n.

Nowe rodziny wielomianów

- Uogólnione wielomiany Bernsteina jednej i wielu zmiennych (2004, 2008)
- Dualne wielomiany Bernsteina różnego typu (2006, 2009)

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Journal of Approximation Theory 138 (2006) 129–150
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Dual generalized Bernstein basis

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Institute of Computer Science, University of Wrocław, ul. Przesmyckiego 20, 51-151 Wrocław, Poland

Received 29 October 2004; accepted 19 October 2005

Communicated by Dany Leviatan
Available online 28 December 2005

Abstract

The generalized Bernstein basis in the space Π_n of polynomials of degree at most n, being an extension of the q-Bernstein basis introduced by Philips [Bernstein polynomials based on the q-integers, Ann. Numer. Math. 4 (1997) 511–518], is given by the formula [S. Lewanowicz, P. Woźny, Generalized Bernstein polynomials, BIT Numer. Math. 44 (2004) 63–78]

$$B_i^n(x;\omega|q) := \frac{1}{(\omega;q)_n}\begin{bmatrix} n \\ i \end{bmatrix}_q x^i(\omega x^{-1};q)_i(x;q)_{n-i} \quad (i=0,1,\ldots,n).$$

We give explicitly the dual basis functions $D_k^n(x;a,b,\omega|q)$ for the polynomials $B_i^n(x;\omega|q)$, in terms of big q-Jacobi polynomials $P_k(x;a,b,\omega/q;q)$, a and b being parameters; the connection coefficients are evaluations of the q-Hahn polynomials. An inverse formula—relating big q-Jacobi, dual generalized Bernstein, and dual q-Hahn polynomials—is also given. Further, an alternative formula is given, representing the dual polynomial D_j^n $(0 \leqslant j \leqslant n)$ as a linear combination of $\min(j, n-j)+1$ big q-Jacobi polynomials with shifted parameters and argument. Finally, we give a recurrence relation satisfied by D_k^n, as well as an identity which may be seen as an analogue of the extended Marsden's identity [R.N. Goldman, Dual polynomial bases, J. Approx. Theory 79 (1994) 311–346].

Keywords: Generalized Bernstein basis; q-Bernstein basis; Bernstein basis; Discrete Bernstein basis; Dual basis; Big q-Jacobi polynomials; Little q-Jacobi polynomials; Shifted Jacobi polynomials

Nowe rodziny wielomianów

- Uogólnione wielomiany Bernsteina jednej i wielu zmiennych (2004, 2008)
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Keywords: Generalized Bernstein basis; q-Bernstein basis; Bernstein basis; Discrete Bernstein basis; Dual basis; Big q-Jacobi polynomials; Little q-Jacobi polynomials; Shifted Jacobi polynomials

$\Rightarrow$ Szerokie zastosowania w grafice, ale o tym za chwilę. . .

Nowe rodziny wielomianów

- Uogólnione wielomiany Bernsteina jednej i wielu zmiennych (2004, 2008)
- Dualne wielomiany Bernsteina różnego typu (2006, 2009)
- Duże bazowe wielomiany Jacobiego dwu zmiennych (2010, 2013)

Journal of Computational and Applied Mathematics 233 (2010) 1554–1561

Contents lists available at ScienceDirect

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Two-variable orthogonal polynomials of big q-Jacobi type

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Dedicated to Professor Jesús S. Dehesa on the occasion of his 60th birthday

MSC:
33D50
33C50

Keywords:
Bivariate big q-Jacobi polynomial
Orthogonality weight
Three-term relation
Partial q-difference equation

ABSTRACT

A four-parameter family of orthogonal polynomials in two discrete variables is defined for a weight function of basic hypergeometric type. The polynomials, which are expressed in terms of univariate big q-Jacobi polynomials, form an extension of Dunkl's bivariate (little) q-Jacobi polynomials [C.F. Dunkl, Orthogonal polynomials in two variables of q-Hahn and q-Jacobi type, SIAM J. Algebr. Discrete Methods 1 (1980) 137–151]. We prove orthogonality property of the new polynomials, and show that they satisfy a three-term relation in a vector-matrix notation, as well as a second-order partial q-difference equation.

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Structure relations for the bivariate big q-Jacobi polynomials

CrossMark

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ARTICLE INFO

Keywords:
Bivariate big q-Jacobi polynomial
Structure relation

ABSTRACT

We give structure relations for the orthogonal polynomials in two variables, defined by Lewanowicz and Woźny [S. Lewanowicz, P. Woźny, J. Comput. Appl. Math. 233 (2010) 1554–1561]

$$P_{n,k}(x,y;a,b,c,d;q) := P_{n-k}(y;a,bcq^{2k+1},dq^k;q)y^k(dq/y;q)_k P_k(x/y;c,b,d/y;q)\quad (n \geqslant 0;\ k=0,1,\ldots,n)$$

where $q \in (0,1)$, $0 < aq, bq, cq < 1$, $d < 0$, and $P_m(t;\alpha,\beta,\gamma;q)$ are univariate big q-Jacobi polynomials. We discuss in full detail the case of the polynomials $P_{n,k}(x,y;a,b,c,0;q)$, which are closely related to Dunkl's bivariate (little) q-Jacobi polynomials [C.F. Dunkl, SIAM J. Algebra. Discr. Methods 1 (1980) 137–151].

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- Dualne wielomiany Bernsteina różnego typu (2006, 2009)
- Duże bazowe wielomiany Jacobiego dwu zmiennych (2010, 2013)

$$P_{n,k}(x, y; a, b, c, d; q)$$

Nowe rodziny wielomianów

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$P_{n,k}(x, y; a, b, c, d; q)$

$d = 0$

Małe wiel. q-Jacobiego(x,y)
(Dunkl, 1980)

Nowe rodziny wielomianów

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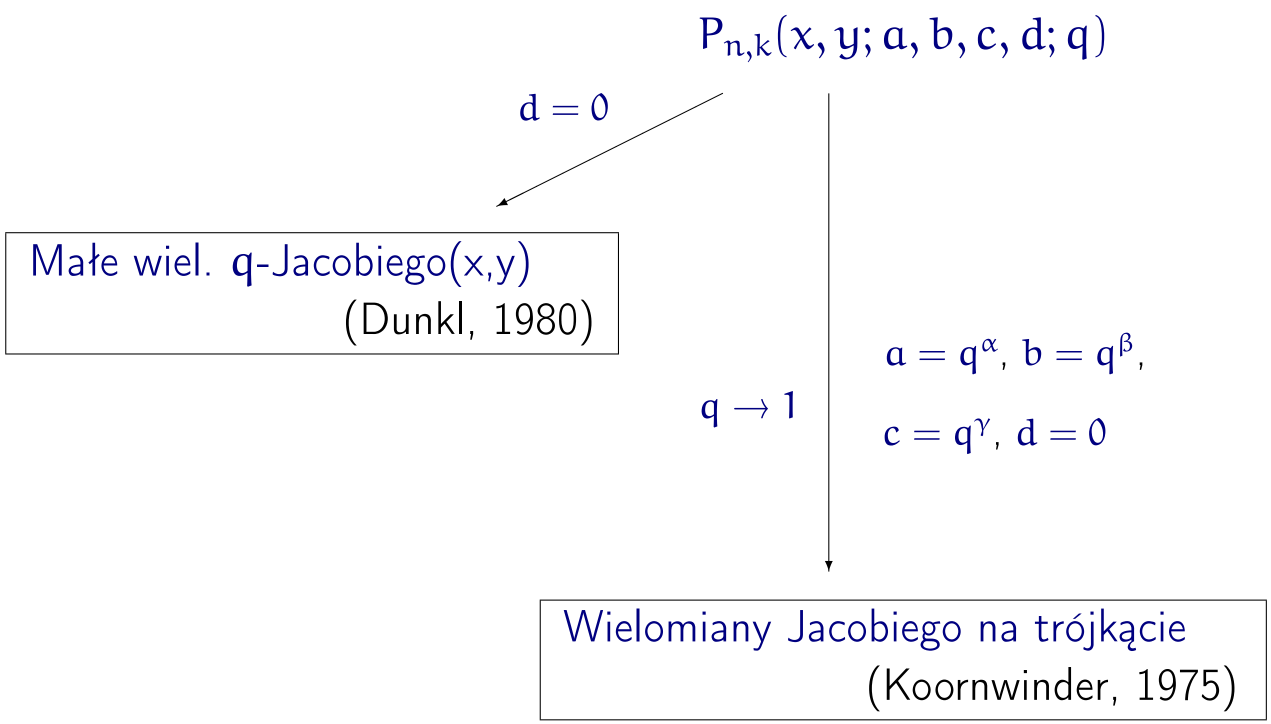

Metody matematyczne modelowania krzywych i powierzchni

- Obniżanie stopnia krzywych i powierzchni Béziera z ograniczeniami (od 2009)

Computer Aided Geometric Design 26 (2009) 566–579

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Computer Aided Geometric Design

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Multi-degree reduction of Bézier curves with constraints, using dual Bernstein basis polynomials

Paweł Woźny, Stanisław Lewanowicz *

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Keywords:
Multi-degree reduction of Bézier curves
Constrained dual Bernstein basis
Dual discrete Bernstein basis

ABSTRACT

We present a novel approach to the problem of multi-degree reduction of Bézier curves with constraints, using the dual constrained Bernstein basis polynomials, associated with the Jacobi scalar product. We give properties of these polynomials, including the explicit orthogonal representations, and the degree elevation formula. We show that the coefficients of the latter formula can be expressed in terms of dual discrete Bernstein polynomials. This result plays a crucial role in the presented algorithm for multi-degree reduction of Bézier curves with constraints. If the input and output curves are of degree n and m, respectively, the complexity of the method is $O(nm)$, which seems to be significantly less than complexity of most known algorithms. Examples are given, showing the effectiveness of the algorithm.

Metody matematyczne modelowania krzywych i powierzchni

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Najszybsza znana metoda (2009)

6 miejsce na liście najczęściej cytowanych artykułów opublikowanych w CAGD

Metody matematyczne modelowania krzywych i powierzchni

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Najszybsza znana metoda (2009)

Journal of Computational and Applied Mathematics 235 (2010) 785–804

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Constrained multi-degree reduction of triangular Bézier surfaces using dual Bernstein polynomials

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Bivariate dual discrete Bernstein basis
Bivariate Jacobi polynomials
Bivariate Hahn polynomials

ABSTRACT

We propose a novel approach to the problem of multi-degree reduction of Bézier triangular patches with prescribed boundary control points. We observe that the solution can be given in terms of bivariate dual discrete Bernstein polynomials. The algorithm is very efficient thanks to using the recursive properties of these polynomials. The complexity of the method is $\mathcal{O}(n^2m^2)$, n and m being the degrees of the input and output Bézier surfaces, respectively. If the approximation—with appropriate boundary constraints—is performed for each patch of several smoothly joined triangular Bézier surfaces, the result is a composite surface of global C^r continuity with a prescribed order r. Some illustrative examples are given.

Najszybsza znana metoda (2010)

Metody matematyczne modelowania krzywych i powierzchni

- Obniżanie stopnia krzywych i powierzchni Béziera z ograniczeniami (od 2009)

Computer Aided Geometric Design 26 (2009) 566–579

Contents lists available at ScienceDirect

Computer Aided Geometric Design

www.elsevier.com/locate/cagd

Multi-degree reduction of Bézier curves with constraints, using dual Bernstein basis polynomials

Paweł Woźny, Stanisław Lewanowicz *

Institute of Computer Science, University of Wrocław, ul. Joliot-Curie 15, 50-383 Wrocław, Poland

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Available online 7 February 2009

Keywords:
Multi-degree reduction of Bézier curves
Constrained dual Bernstein basis
Dual discrete Bernstein basis

ABSTRACT

We present a no
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with the Jacobi
explicit orthogon
coefficients of th
polynomials. This
reduction of Bé
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significantly less
the effectiveness

Najszybsza znana metoda (2009)

Journal of Computational and Applied Mathematics 235 (2010) 785–804

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Constrained multi-degree reduction of triangular Bézier surfaces using dual Bernstein polynomials

Paweł Woźny, Stanisław Lewanowicz *

Institute of Computer Science, University of Wrocław, ul. F. Joliot-Curie 15, 50-383 Wrocław, Poland

ARTICLE INFO

ABSTRACT

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Najszybsza znana metoda (2010)

Applied Mathematics and Computation 217 (2011) 4596–4611

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Multi-degree reduction of tensor product Bézier surfaces with general boundary constraints

Stanisław Lewanowicz *, Paweł Woźny

Institute of Computer Science, University of Wrocław, ul. F. Joliot-Curie 15, 50-383 Wrocław, Poland

ARTICLE INFO

Keywords:
Rectangular Bézier surface
Multi-degree reduction
Constrained dual Bernstein basis
Jacobi polynomials
Hahn polynomials

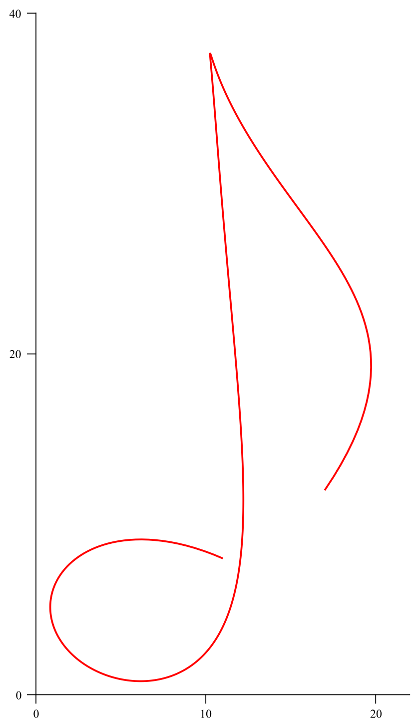
ABSTRACT

We propose an efficient approach to the problem of multi-degree reduction of rectangular Bézier patches, with prescribed boundary control points. We observe that the solution can be given in terms of constrained bivariate dual Bernstein polynomials. The complexity of the method is $\mathcal{O}(mn_1n_2)$ with $m := \min(m_1, m_2)$, where (n_1, n_2) and (m_1, m_2) is the degree of the input and output Bézier surface, respectively. If the approximation—with appropriate boundary constraints—is performed for each patch of several smoothly joined rectangular Bézier surfaces, the result is a composite surface of global C^r continuity with a prescribed $r \geqslant 0$. In the detailed discussion, we restrict ourselves to $r \in \{0, 1\}$, which is the most important case in practical application. Some illustrative examples are given.

Najszybsza znana metoda (2011)

Metody matematyczne modelowania krzywych i powierzchni

- Obniżanie stopnia krzywych i powierzchni Béziera z ograniczeniami (od 2009)



krzywa Béziera stopnia n

Metody matematyczne modelowania krzywych i powierzchni

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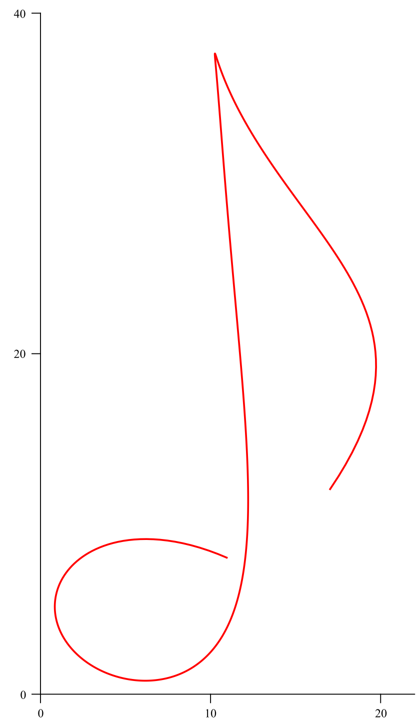
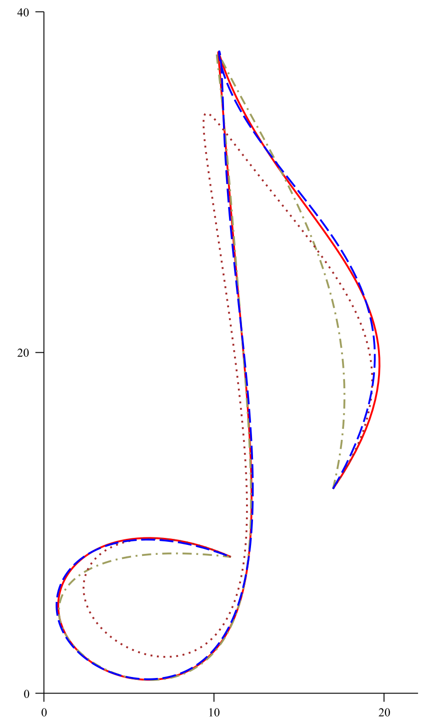


krzywa Béziera stopnia n



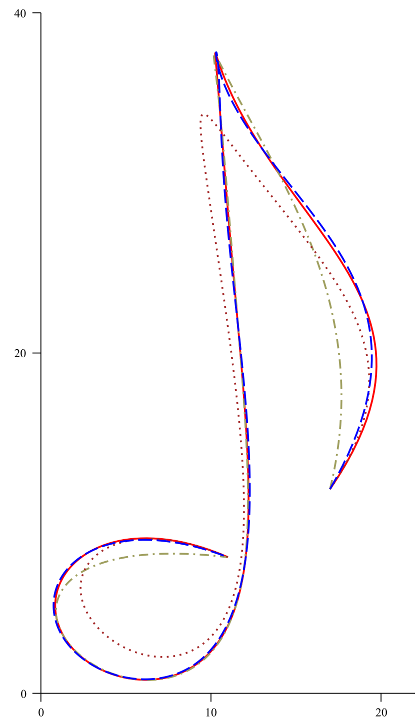
krzywe Béziera stopnia $m < n$

Metody matematyczne modelowania krzywych i powierzchni

- Obniżanie stopnia krzywych i powierzchni Béziera z ograniczeniami (od 2009)

krzywa Béziera stopnia n

$\Longrightarrow$



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$\longleftarrow$ Niebieska najbliżej!

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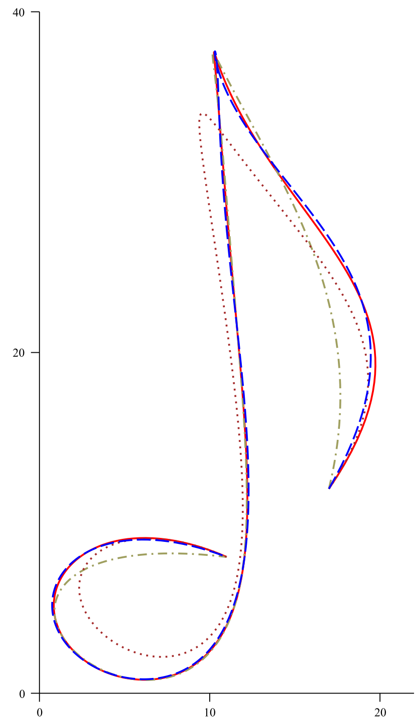

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Minimalizacja odległości średniokwadratowej

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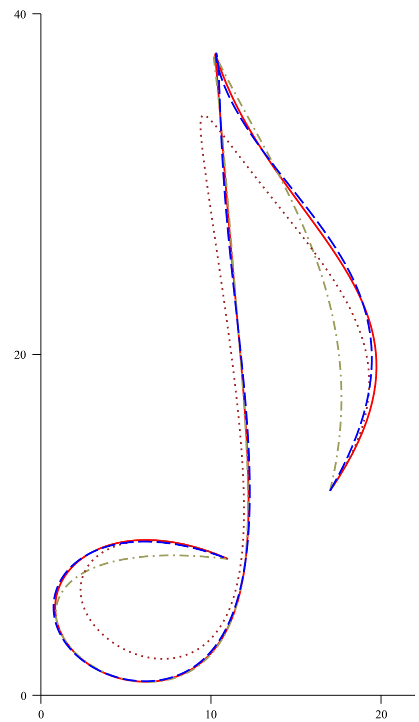

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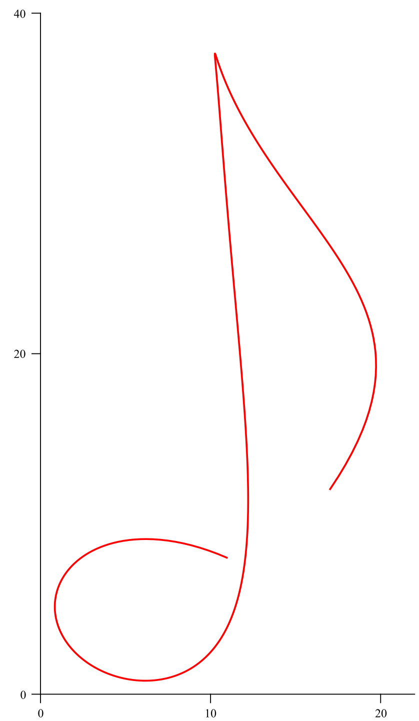

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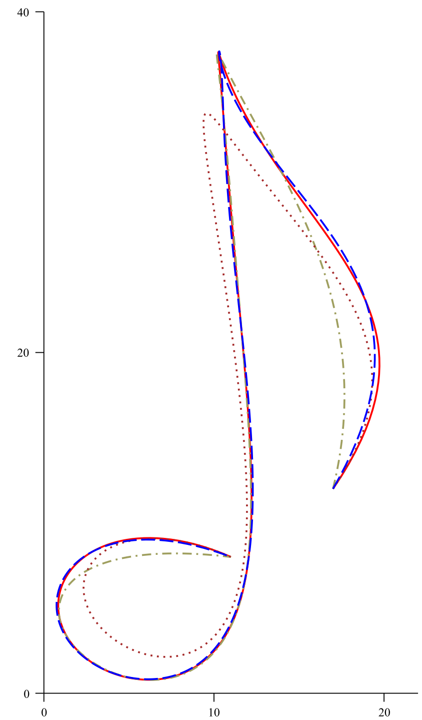

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Minimalizacja odległości średniokwadratowej:

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- zastosowanie dualnych wielomianów Bernsteina: $O(nm)$

Metody matematyczne modelowania krzywych i powierzchni

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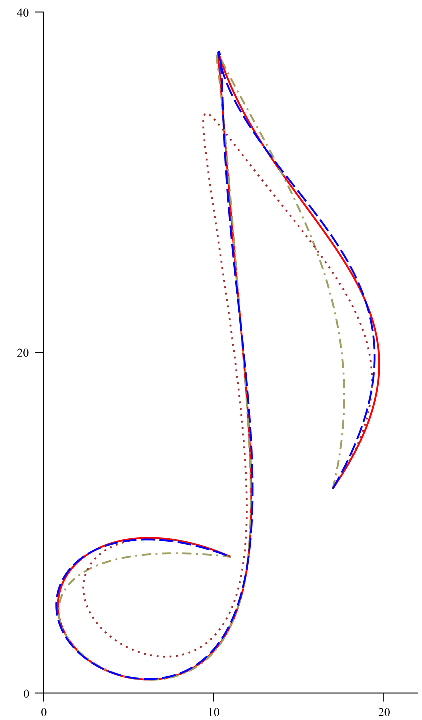
krzywa Béziera stopnia n

$\Longrightarrow$



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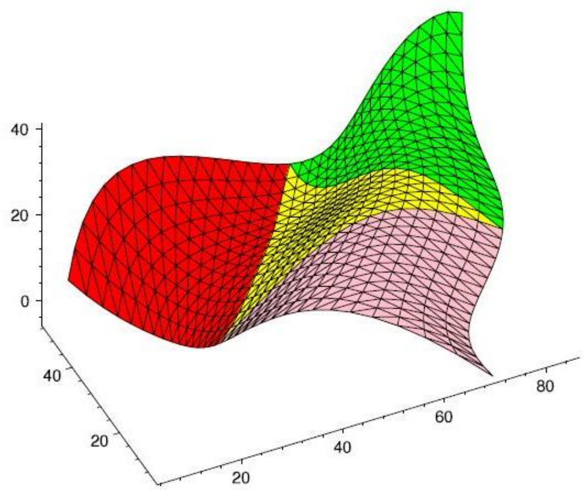
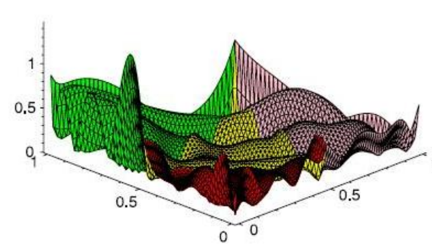


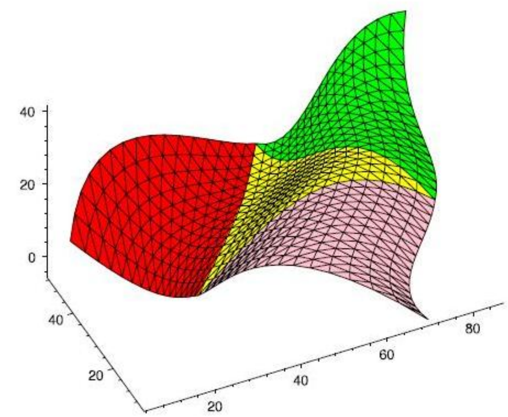
Cztery płaty stopnia 7 = 112 pkt. kontrolnych

Kompresja: ok. 4,7

$\Longrightarrow$



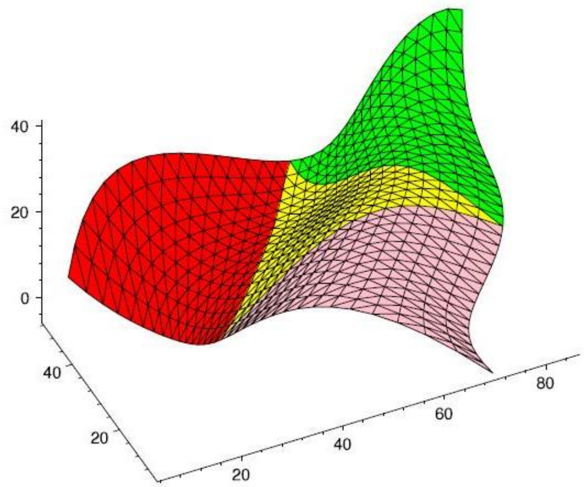
Błąd



Cztery płaty stopnia 3 = 24 pkt. kontrolne

Metody matematyczne modelowania krzywych i powierzchni

- Obniżanie stopnia krzywych i powierzchni Béziera z ograniczeniami (od 2009)



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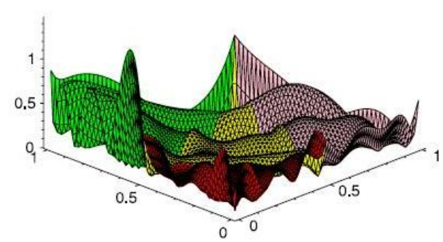

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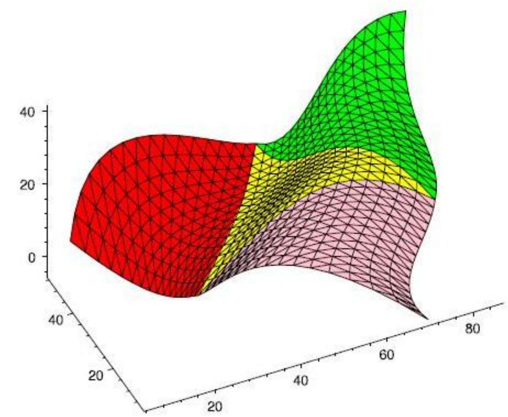

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Minimalizacja odległości średniokwadratowej

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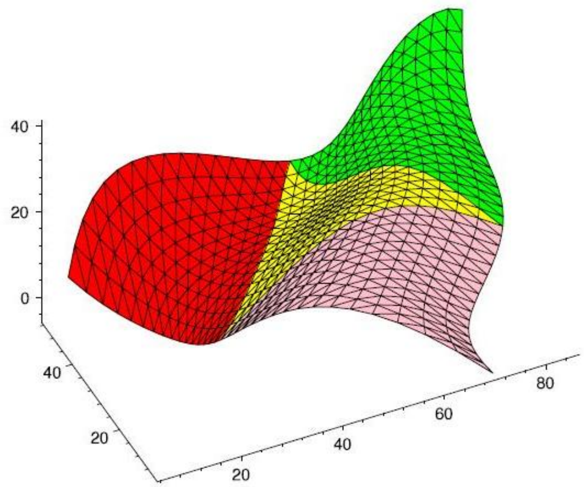

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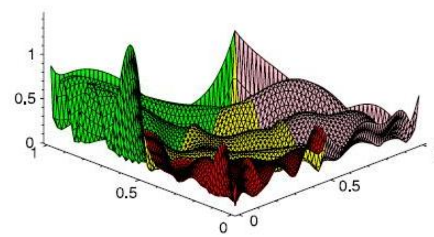

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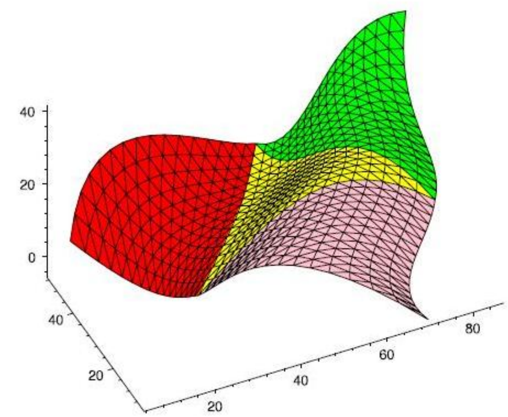

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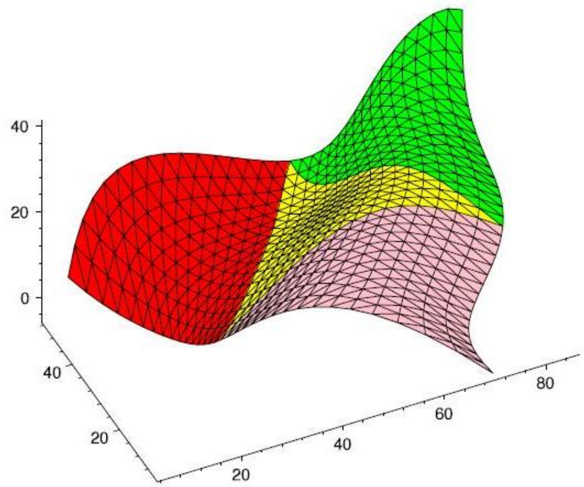

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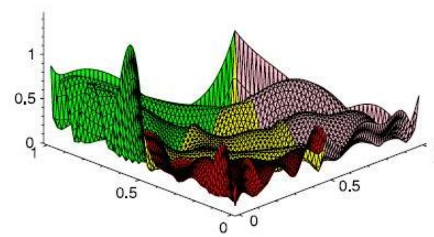

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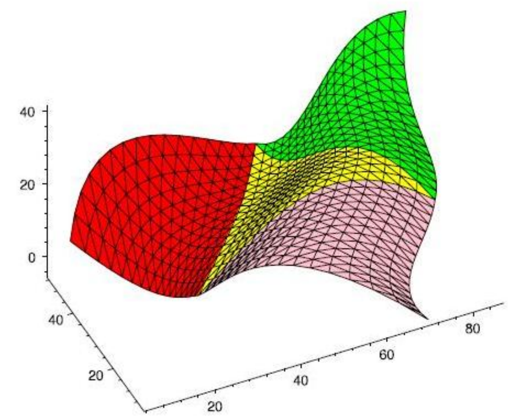

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Minimalizacja odległości średniokwadratowej:

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Metody matematyczne modelowania krzywych i powierzchni

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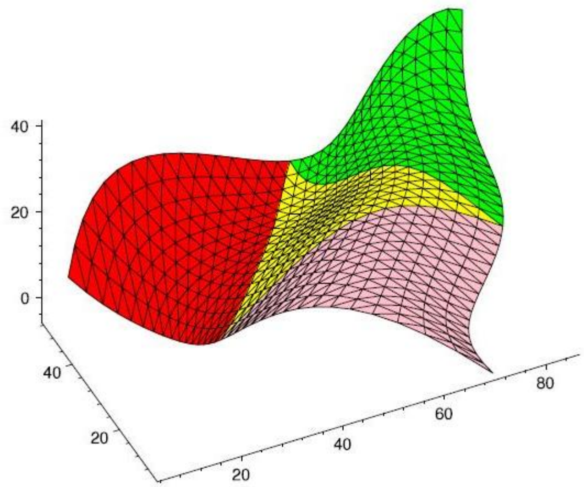

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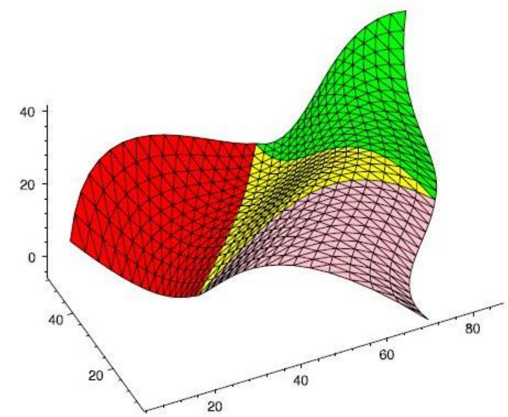

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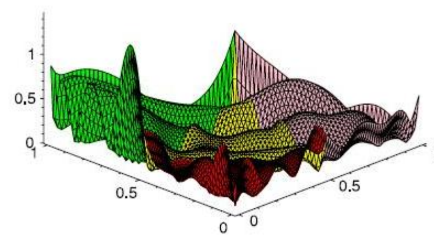

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Metody matematyczne modelowania krzywych i powierzchni

- Obniżanie stopnia krzywych i powierzchni Béziera z ograniczeniami (od 2009)
- Aproksymacja wymiernych krzywych Béziera wielomianowymi krzywymi Béziera (2012)

Numer Algor (2012) 59:607–622
DOI 10.1007/s11075-011-9507-0

ORIGINAL PAPER

Polynomial approximation of rational Bézier curves with constraints

Stanisław Lewanowicz · Paweł Woźny · Paweł Keller

Received: 31 May 2011 / Accepted: 20 September 2011 /
Published online: 16 October 2011

Abstract We present an efficient method to solve the problem of the constrained least squares approximation of the rational Bézier curve by the polynomial Bézier curve. The presented algorithm uses the dual constrained Bernstein basis polynomials, and exploits their recursive properties. Examples are given, showing the effectiveness of the algorithm.

Keywords Rational Bézier curve · Polynomial approximation · Constrained dual Bernstein basis

Najszybsza znana metoda

Metody matematyczne modelowania krzywych i powierzchni

- Obniżanie stopnia krzywych i powierzchni Béziera z ograniczeniami (od 2009)
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- Scalanie krzywych Béziera (2014)

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- Scalanie krzywych Béziera (2014) $\Leftarrow$ Nowość. Najszybsza znana metoda

Metody matematyczne modelowania krzywych i powierzchni

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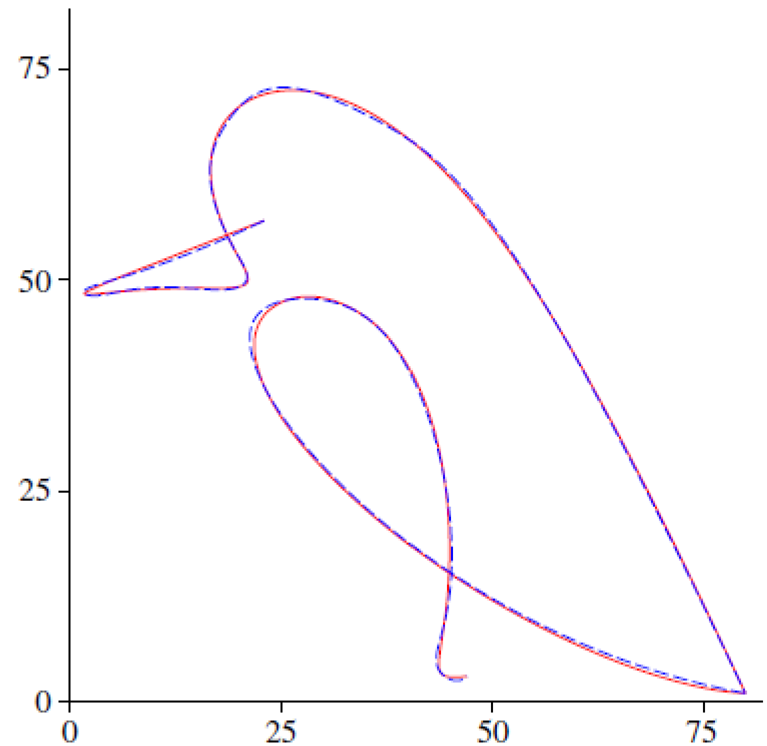
- Scalanie krzywych Béziera (2014)

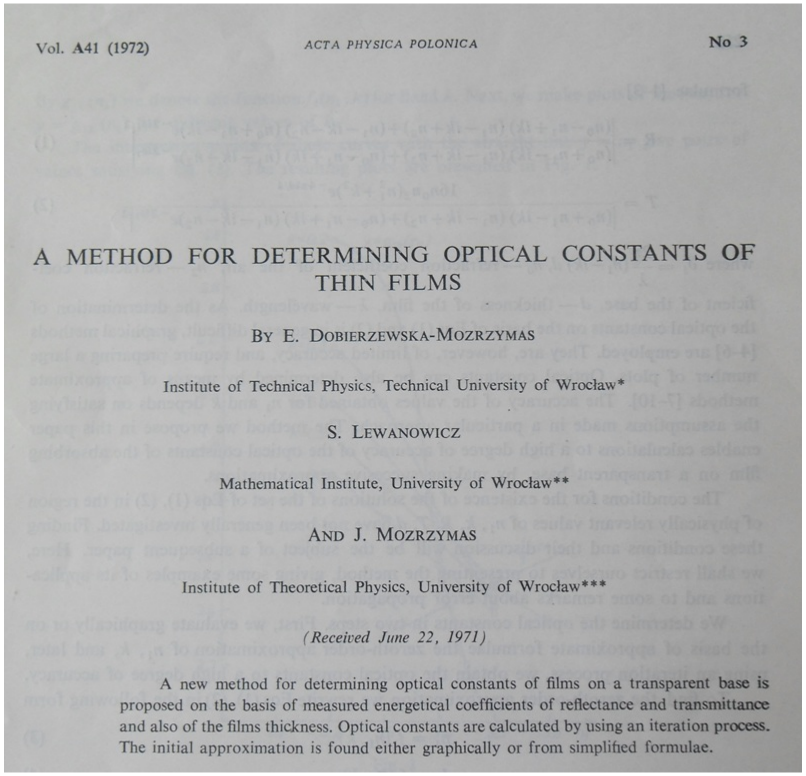
Zastosowania praktyczne:

- kompresja danych
- wymiana danych pomiędzy różnymi systemami CAG
- większa swoboda działania projektantów i grafików



Metody numeryczne, czyli wracamy do początków. . .

- Zastosowania metod numerycznych w optyce i medycynie (1972, 1974)



Vol. A41 (1972) ACTA PHYSICA POLONICA No 3

A METHOD FOR DETERMINING OPTICAL CONSTANTS OF THIN FILMS

BY E. DOBIERZEWSKA-MOZRZYMAS

Institute of Technical Physics, Technical University of Wrocław*

S. LEWANOWICZ

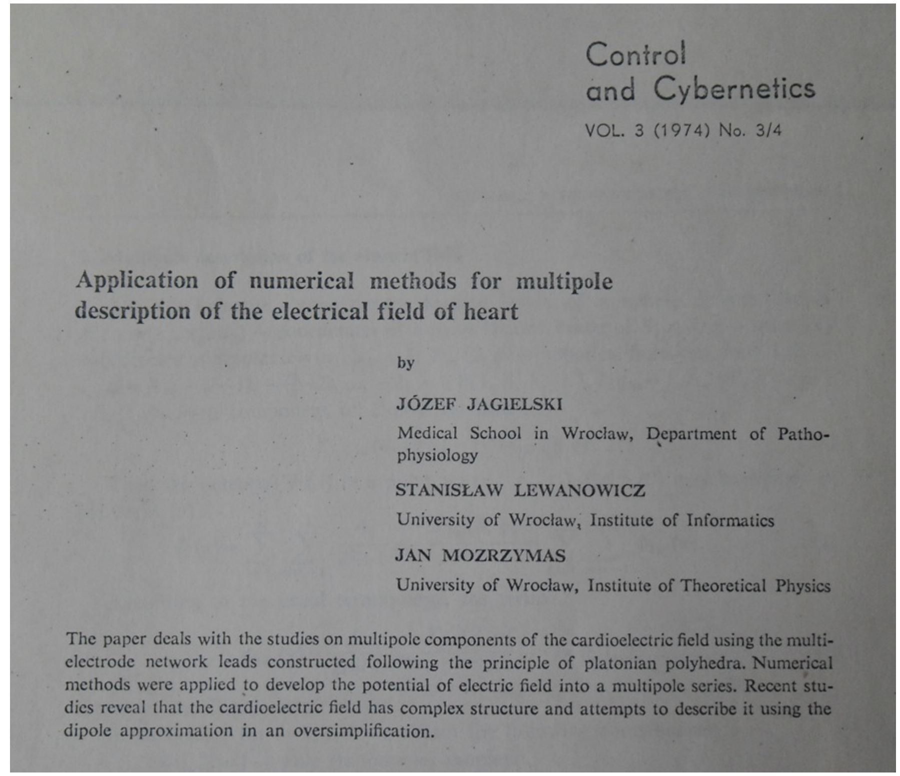
Mathematical Institute, University of Wrocław**

AND J. MOZRZYMAS

Institute of Theoretical Physics, University of Wrocław***

(*Received June 22, 1971*)

A new method for determining optical constants of films on a transparent base is proposed on the basis of measured energetical coefficients of reflectance and transmittance and also of the films thickness. Optical constants are calculated by using an iteration process. The initial approximation is found either graphically or from simplified formulae.



Control and Cybernetics

VOL. 3 (1974) No. 3/4

Application of numerical methods for multipole description of the electrical field of heart

by

JÓZEF JAGIELSKI

Medical School in Wrocław, Department of Pathophysiology

STANISŁAW LEWANOWICZ

University of Wrocław, Institute of Informatics

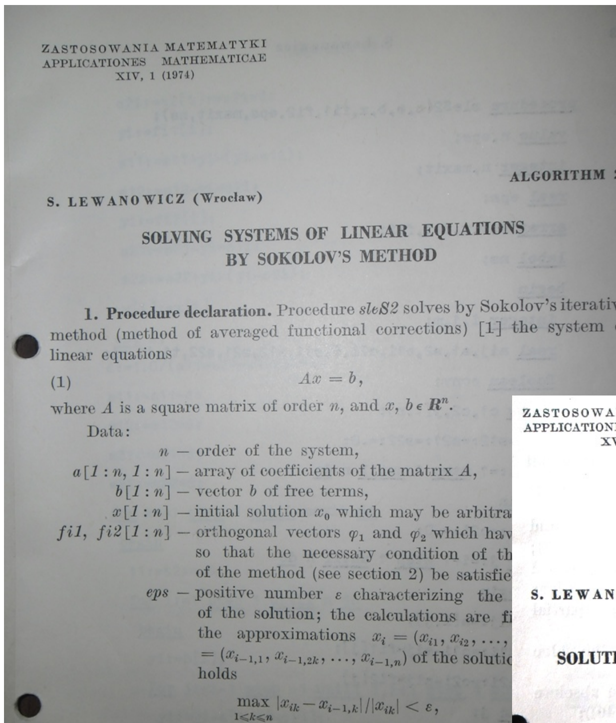
JAN MOZRZYMAS

University of Wrocław, Institute of Theoretical Physics

The paper deals with the studies on multipole components of the cardioelectric field using the multielectrode network leads constructed following the principle of platonian polyhedra. Numerical methods were applied to develop the potential of electric field into a multipole series. Recent studies reveal that the cardioelectric field has complex structure and attempts to describe it using the dipole approximation in an oversimplification.

Metody numeryczne, czyli wracamy do początków. . .

- Zastosowania metod numerycznych w optyce i medycynie (1972, 1974)
- Implementacja algorytmów numerycznych dla Odry 1204 (1974, 1976)



ZASTOSOWANIA MATEMATYKI
APPLICATIONES MATHEMATICAE
XIV, 1 (1974)

ALGORITHM 2

S. LEWANOWICZ (Wrocław)

SOLVING SYSTEMS OF LINEAR EQUATIONS
BY SOKOLOV'S METHOD

1. Procedure declaration. Procedure *sleS2* solves by Sokolov's iterativ
method (method of averaged functional corrections) [1] the system o
linear equations

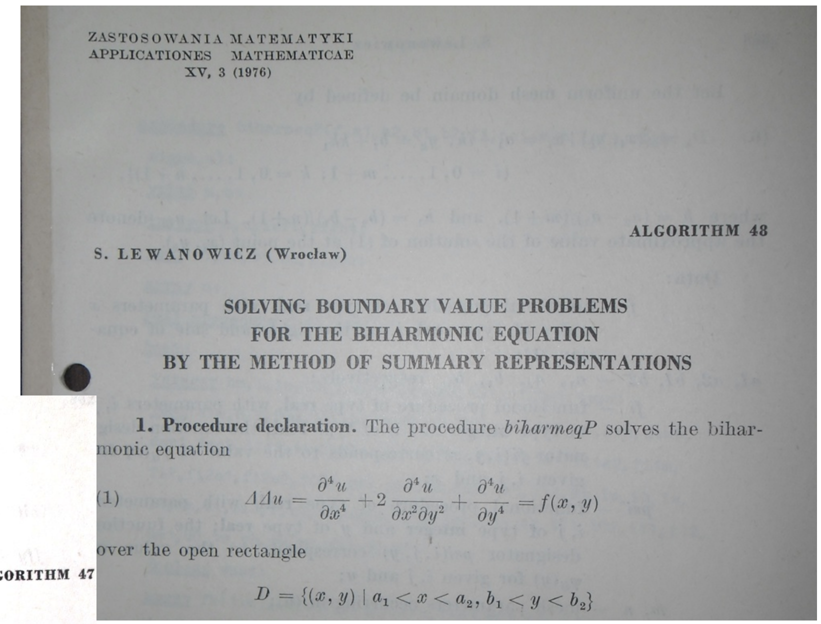
(1) $Ax = b,$

where A is a square matrix of order n, and $x, b \in R^n$.

Data:

n — order of the system,
$a[1:n, 1:n]$ — array of coefficients of the matrix A,
$b[1:n]$ — vector b of free terms,
$x[1:n]$ — initial solution x_0 which may be arbitra
$fi1, fi2[1:n]$ — orthogonal vectors φ_1 and φ_2 which hav
so that the necessary condition of th
of the method (see section 2) be satisfie
eps — positive number ε characterizing the
of the solution; the calculations are fi
the approximations $x_i = (x_{i1}, x_{i2}, \ldots,$
$= (x_{i-1,1}, x_{i-1,2k}, \ldots, x_{i-1,n})$ of the solutic
holds

$$\max_{1 \leqslant k \leqslant n} |x_{ik} - x_{i-1,k}|/|x_{ik}| < \varepsilon,$$



ZASTOSOWANIA MATEMATYKI
APPLICATIONES MATHEMATICAE
XV, 3 (1976)

ALGORITHM 48

S. LEWANOWICZ (Wrocław)

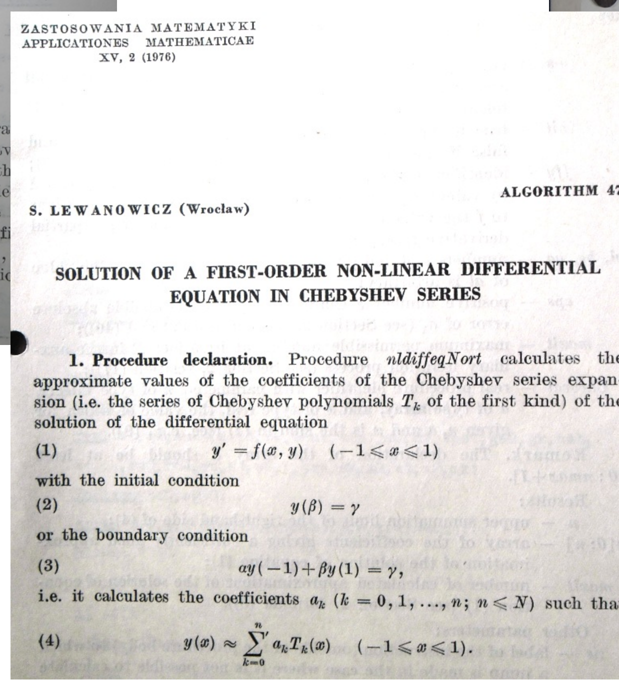
SOLVING BOUNDARY VALUE PROBLEMS
FOR THE BIHARMONIC EQUATION
BY THE METHOD OF SUMMARY REPRESENTATIONS

1. Procedure declaration. The procedure *biharmeqP* solves the biharmonic equation

(1) $$\Delta\Delta u = \frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = f(x, y)$$

over the open rectangle

$$D = \{(x, y) \mid a_1 < x < a_2,\, b_1 < y < b_2\}$$



ZASTOSOWANIA MATEMATYKI
APPLICATIONES MATHEMATICAE
XV, 2 (1976)

ALGORITHM 47

S. LEWANOWICZ (Wrocław)

SOLUTION OF A FIRST-ORDER NON-LINEAR DIFFERENTIAL
EQUATION IN CHEBYSHEV SERIES

1. Procedure declaration. Procedure *nldiffeqNort* calculates the approximate values of the coefficients of the Chebyshev series expansion (i.e. the series of Chebyshev polynomials T_k of the first kind) of the solution of the differential equation

(1) $y' = f(x, y) \quad (-1 \leqslant x \leqslant 1)$

with the initial condition

(2) $y(\beta) = \gamma$

or the boundary condition

(3) $\alpha y(-1) + \beta y(1) = \gamma,$

i.e. it calculates the coefficients a_k $(k = 0, 1, \ldots, n;\ n \leqslant N)$ such that

(4) $$y(x) \approx \sum_{k=0}^{n}{}' a_k T_k(x) \quad (-1 \leqslant x \leqslant 1).$$

Metody numeryczne, czyli wracamy do początków...

- Zastosowania metod numerycznych w optyce i medycynie (1972, 1974)
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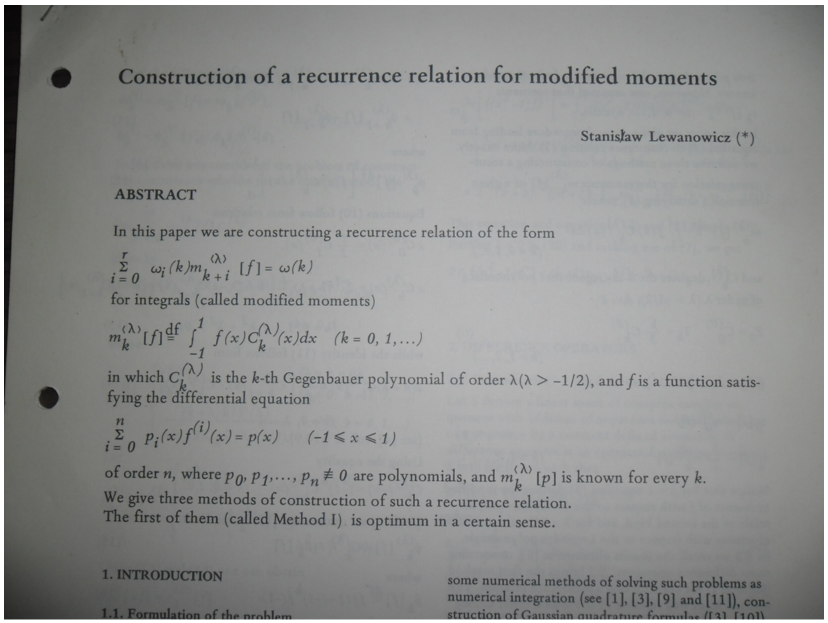


Construction of a recurrence relation for modified moments

Stanisław Lewanowicz (*)

ABSTRACT

In this paper we are constructing a recurrence relation of the form

$$\sum_{i=0}^{r} \omega_i(k) m_{k+i}^{\langle\lambda\rangle} [f] = \omega(k)$$

for integrals (called modified moments)

$$m_k^{\langle\lambda\rangle}[f] \stackrel{\mathrm{df}}{=} \int_{-1}^{1} f(x) C_k^{(\lambda)}(x) dx \quad (k = 0, 1, \ldots)$$

in which $C_k^{(\lambda)}$ is the k-th Gegenbauer polynomial of order $\lambda (\lambda > -1/2)$, and f is a function satisfying the differential equation

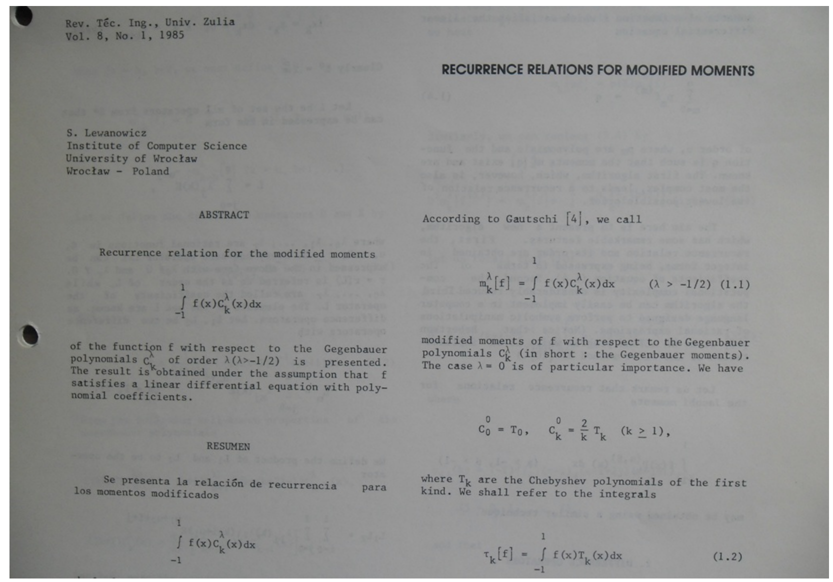
$$\sum_{i=0}^{n} p_i(x) f^{(i)}(x) = p(x) \qquad (-1 \leqslant x \leqslant 1)$$

of order n, where $p_0, p_1, \ldots, p_n \not\equiv 0$ are polynomials, and $m_k^{\langle\lambda\rangle}[p]$ is known for every k.
We give three methods of construction of such a recurrence relation.
The first of them (called Method I) is optimum in a certain sense.

1. INTRODUCTION

1.1. Formulation of the problem

some numerical methods of solving such problems as numerical integration (see [1], [3], [9] and [11]), construction of Gaussian quadrature formulas ([3], [10])



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Vol. 8, No. 1, 1985

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University of Wrocław
Wrocław - Poland

ABSTRACT

Recurrence relation for the modified moments

$$\int_{-1}^{1} f(x) C_k^{\lambda}(x) dx$$

of the function f with respect to the Gegenbauer polynomials C_k^{λ} of order $\lambda (\lambda > -1/2)$ is presented. The result is obtained under the assumption that f satisfies a linear differential equation with polynomial coefficients.

RESUMEN

Se presenta la relación de recurrencia para los momentos modificados

$$\int_{-1}^{1} f(x) C_k^{\lambda}(x) dx$$

RECURRENCE RELATIONS FOR MODIFIED MOMENTS

According to Gautschi [4], we call

$$m_k^{\lambda}[f] = \int_{-1}^{1} f(x) C_k^{\lambda}(x) dx \qquad (\lambda > -1/2) \quad (1.1)$$

modified moments of f with respect to the Gegenbauer polynomials C_k^{λ} (in short : the Gegenbauer moments). The case $\lambda = 0$ is of particular importance. We have

$$C_0^0 = T_0, \quad C_k^0 = \frac{2}{k} T_k \quad (k \geq 1),$$

where T_k are the Chebyshev polynomials of the first kind. We shall refer to the integrals

$$\tau_k[f] = \int_{-1}^{1} f(x) T_k(x) dx \qquad (1.2)$$

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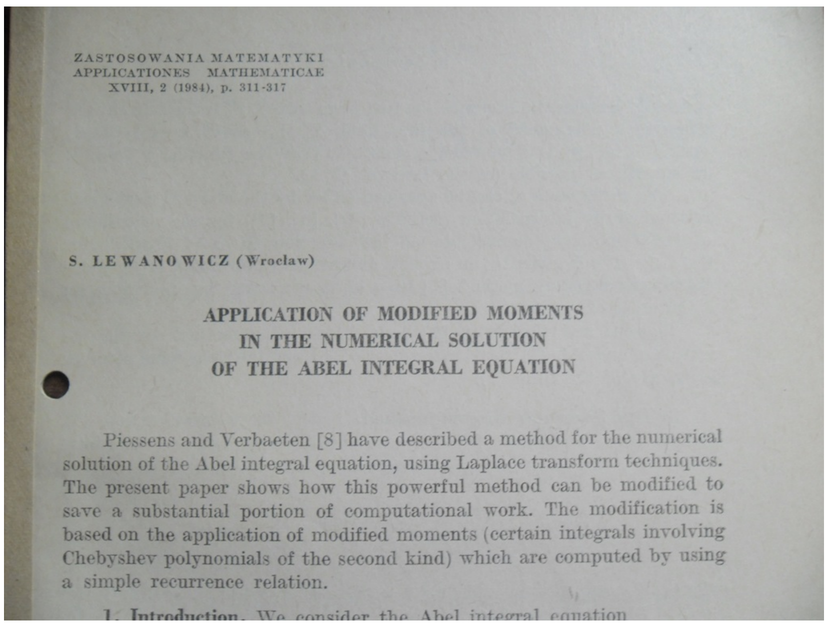
- Numeryczne rozwiązywanie równania całkowego Abela (1984, 1990)

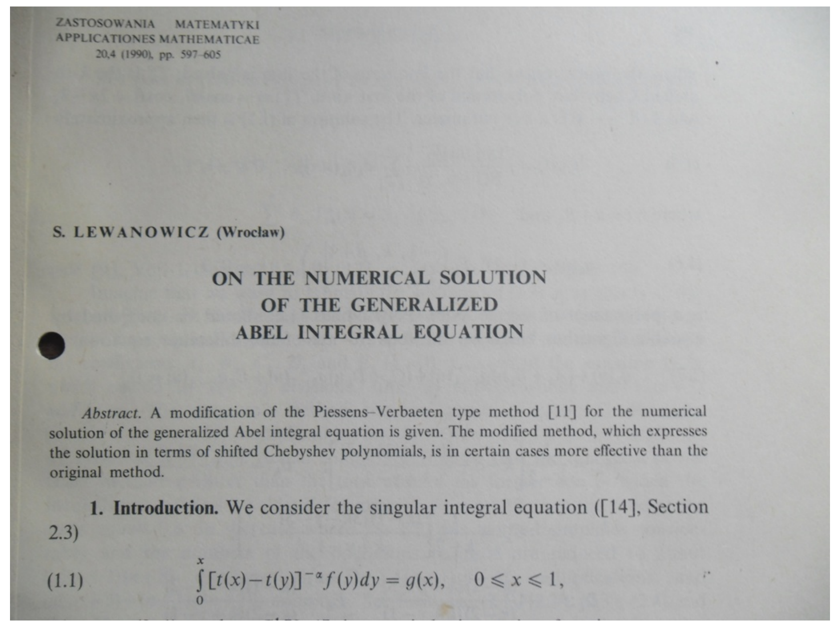

ZASTOSOWANIA MATEMATYKI
APPLICATIONES MATHEMATICAE
XVIII, 2 (1984), p. 311-317

S. LEWANOWICZ (Wrocław)

APPLICATION OF MODIFIED MOMENTS
IN THE NUMERICAL SOLUTION
OF THE ABEL INTEGRAL EQUATION

Piessens and Verbaeten [8] have described a method for the numerical solution of the Abel integral equation, using Laplace transform techniques. The present paper shows how this powerful method can be modified to save a substantial portion of computational work. The modification is based on the application of modified moments (certain integrals involving Chebyshev polynomials of the second kind) which are computed by using a simple recurrence relation.

1. Introduction. We consider the Abel integral equation


ZASTOSOWANIA MATEMATYKI
APPLICATIONES MATHEMATICAE
20,4 (1990), pp. 597-605

S. LEWANOWICZ (Wrocław)

ON THE NUMERICAL SOLUTION
OF THE GENERALIZED
ABEL INTEGRAL EQUATION

Abstract. A modification of the Piessens-Verbaeten type method [11] for the numerical solution of the generalized Abel integral equation is given. The modified method, which expresses the solution in terms of shifted Chebyshev polynomials, is in certain cases more effective than the original method.

1. Introduction. We consider the singular integral equation ([14], Section 2.3)

(1.1) $$\int_0^x [t(x)-t(y)]^{-\alpha} f(y)\,dy = g(x), \quad 0 \leqslant x \leqslant 1,$$

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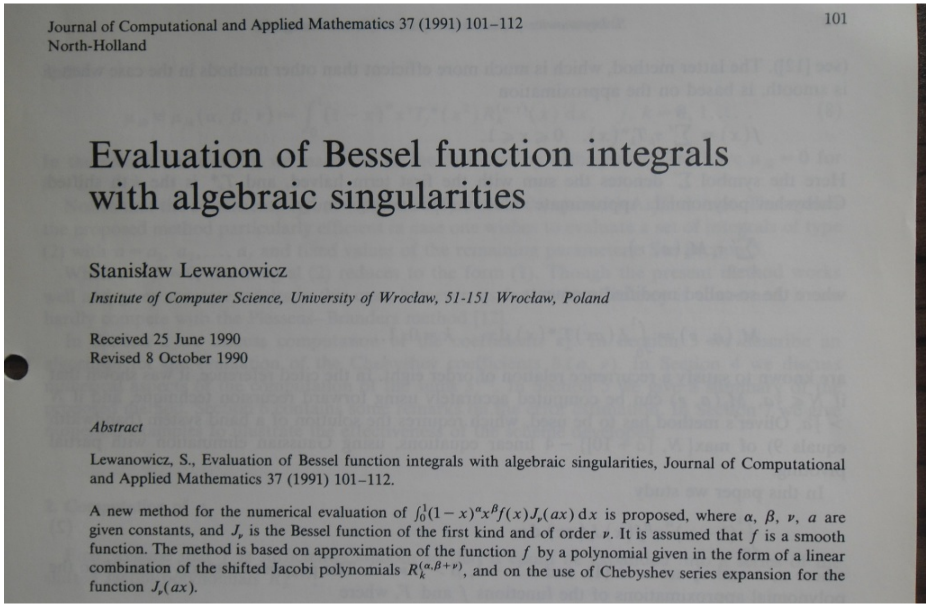
- Numeryczne rozwiązywanie równania całkowego Abela (1984, 1990)
- Numeryczne obliczanie całek postaci $\int_0^1 (1-x)^\alpha x^\beta f(x) J_\nu(ax)\,dx$ (1991)


Journal of Computational and Applied Mathematics 37 (1991) 101–112
North-Holland
101

Evaluation of Bessel function integrals with algebraic singularities

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Abstract

Lewanowicz, S., Evaluation of Bessel function integrals with algebraic singularities, Journal of Computational and Applied Mathematics 37 (1991) 101–112.

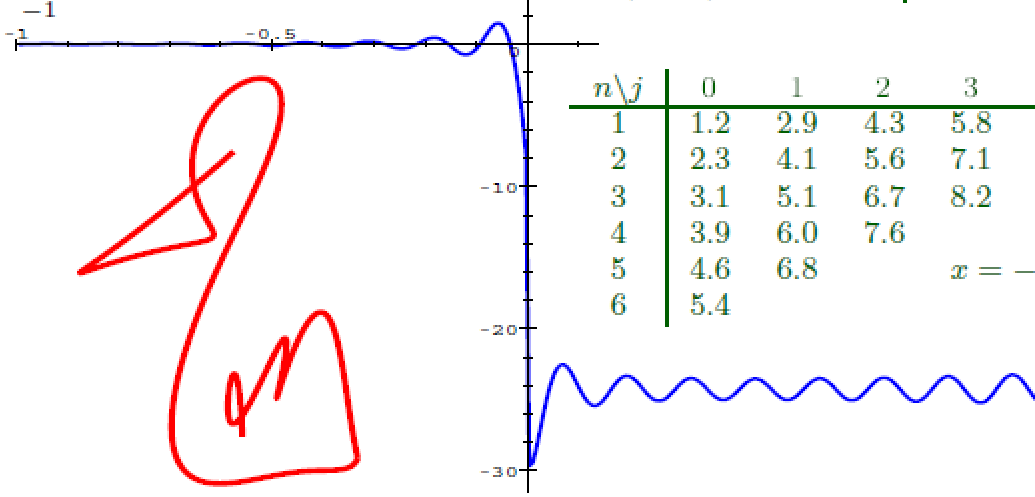
A new method for the numerical evaluation of $\int_0^1 (1-x)^\alpha x^\beta f(x) J_\nu(ax)\,dx$ is proposed, where α, β, ν, a are given constants, and J_ν is the Bessel function of the first kind and of order ν. It is assumed that f is a smooth function. The method is based on approximation of the function f by a polynomial given in the form of a linear combination of the shifted Jacobi polynomials $R_k^{(\alpha,\beta+\nu)}$, and on the use of Chebyshev series expansion for the function $J_\nu(ax)$.

Paweł Keller | Paweł Woźny | Rafał Nowak

Paweł Keller — Paweł Woźny — Rafał Nowak

$$\int_{-1}^{y} e^{2(1+t)}|t|^{-0.9}\sin\big(50(1+t)\big)\,dt$$

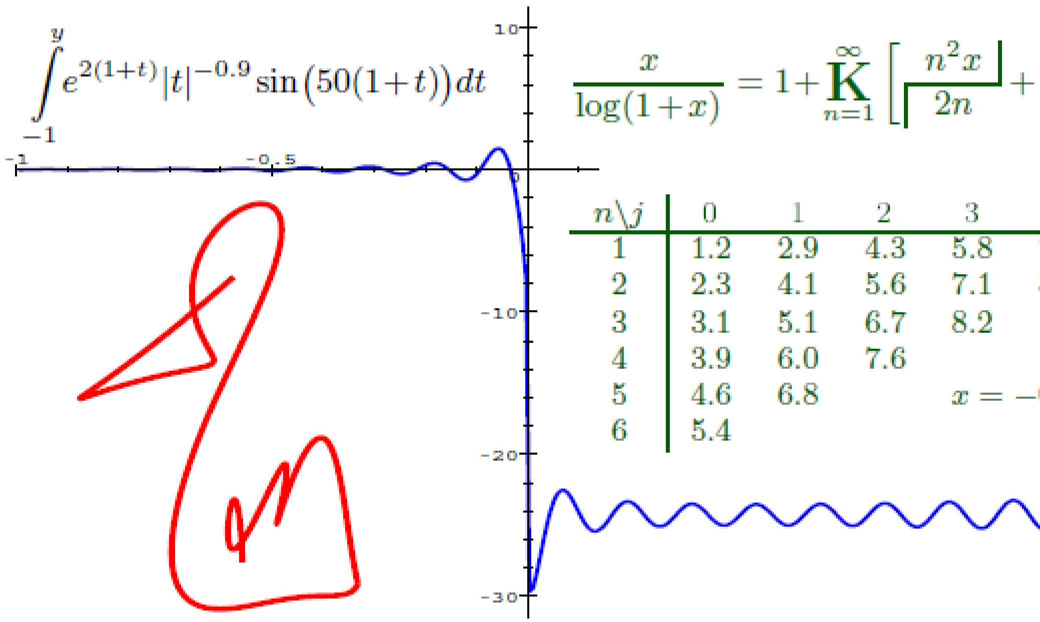
$$\frac{x}{\log(1+x)} = 1+\mathop{\mathrm{K}}_{n=1}^{\infty}\left[\frac{n^2x}{2n}+\frac{n^2x}{2n+1}\right]$$



$n\backslash j$	0	1	2	3	4	5
1	1.2	2.9	4.3	5.8	7.2	8.7
2	2.3	4.1	5.6	7.1	8.6	
3	3.1	5.1	6.7	8.2		
4	3.9	6.0	7.6			
5	4.6	6.8				
6	5.4					

$x=-0.9$

Paweł Keller | Paweł Woźny | Rafał Nowak



$$\frac{x}{\log(1+x)} = 1+\mathop{\mathrm{K}}_{n=1}^{\infty}\left[\frac{n^2x}{2n}+\frac{n^2x}{2n+1}\right]$$

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5	4.6	6.8		$x=-0.9$		
6	5.4					

Wszystkiego najlepszego Panie Profesorze!

¡Saludos desde Vigo!